Astron. Obs.
MAR 1 1917
Univ. of Michigan

VOLUME XLV NUMBER 1

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Solar Observatory of the Carnegie Institution of Washington

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JANUARY 1917

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THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

Agents
THE CAMBRIDGE UNIVERSITY PRESS, London and Edinburgh
KARL W. HIERSEMANN, Leipsig
THE MARUZEN-KABUSHIKI-KAISHA, Tokyo, Osaka, Kyoto, Fukuoka, Sendai
THE MISSION BOOK COMPANY, Shanghai

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The Astrophysical Journal is published by the University of Chicago at the University Press during each month except February and August. ¶ The subscription price is $5.00 a year; the price of single copies is 65 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶ Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, Shanghai. ¶ Postage is charged extra as follows: For Canada, 30 cents on annual subscriptions (total $5.30), on single copies, 3 cents (total 68 cents); for all other countries in the Postal Union, 62 cents on annual subscriptions (total $5.62), on single copies, 11 cents (total 76 cents). ¶ Patrons are requested to make all remittances payable to The University of Chicago Press in postal or express money orders or bank drafts.

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Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when they have been lost in transit.

Business correspondence should be addressed to The University of Chicago Press, Chicago, Ill.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address of the Editorial Office is "Arcturus, Chicago."

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the act of March 3, 1879.

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THE
ASTROPHYSICAL JOURNAL
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AND ASTRONOMICAL PHYSICS

VOLUME XLV JANUARY 1917 NUMBER 1

THE PHYSICAL INTERPRETATION OF ALBEDO. I

By LOUIS BELL

The albedo of the bodies in the solar system is one of the few clues available for the analysis of their physical characteristics. The values which have been current for albedo have unfortunately been, for various reasons, exceedingly misleading, and prior to the publication of H. N. Russell's important paper[1] there were no published values of much service in furthering physical investigation. In undertaking an examination of the subject some two years since the writer was compelled to make some drastic revisions of the current values, which Russell's contribution has made it possible to get into sufficiently satisfactory form for the purpose in hand. Primarily, the chief trouble in straightening out the matter of albedo has arisen from the fact that the term itself is used in different senses by different writers and sometimes by the same writer.

There are three quantities which have traveled under the general name of albedo as regards the heavenly bodies. First stands what may properly be called the *geometric albedo*, which is merely the proportion of the incident solar light visibly reflected from the given body at full phase. This takes no account of any fact or hypothesis regarding the relation of the reflecting power to the shape of the

[1] *Astrophysical Journal*, 43, 101, 173, 1916.

body, or of any law connecting the reflecting power of its surface with the angles of incidence and reflection. It depends merely on the geometrical relations of the body to the sun and earth, and on its photometric ratio to the sun. This is Russell's quantity p, which, as he indicates, is important in comparing the albedo thus defined with the reflectivities of terrestrial substances. It is also significant in giving, when compared with these reflectivities, a clue to the character of the surface.

Unhappily, this quantity, which seems very direct and obvious, has often been confused with an entirely different class of quantities which may properly be called *spherical albedos* and which have to do with the proportion of visible reflected solar light, taking into account the sphericity of the body and the known or assumed relations between the reflectivity and the angles of incidence and of reflection.

Of such quantities there are as many as there may be hypotheses assumed regarding incidence and reflection. The best known of the class is what should properly be called Lambert's albedo, based upon his cosine law as applied to a perfectly diffusing sphere. It is this which has been most commonly known as "albedo" in astronomical literature. Numerically it is $3/2$ the geometric albedo of such a perfectly diffusing sphere, and physically it is the reflectivity of an element of such surface at substantially normal incidence, as Lambert himself indicates in the following words: "Porro eodem hoc casu, quo nempe planetae pleno orbe lucent, claritas totius disci media est $2/3$ claritatis centralis."[1]

Euler's law of reflection,[2] has occasionally been used for reckoning a value of albedo which should be carefully distinguished both from Lambert's and from others. The relation of any kind of spherical albedo to the apparent geometric albedo obviously depends on the law assumed or found for the spherical case. Numerically the spherical albedo may turn out greater than the geometric, as by the assumed laws just noticed, or less, as in some cases of Bond's albedo, *q.v.*

Another hypothetical law of reflection, the Lommel-Seeliger law, which Russell shows (*loc. cit.*), leads at times to values of the specific

[1] *Photometria*, § 1084, p. 484. [2] *Astrophysical Journal*, 43, 178, 1916.

albedo impossibly large or even exceeding unity, was put forth by Seeliger[1] and used by Müller[2] for a tentative series of albedo values. Finally, Russell (*loc. cit.*), abandoning hypothesis in favor of experiment, has recurred to Bond's definition of albedo as the ratio of the whole amount of light reflected from the sphere to the whole amount incident on it, and has tabulated these values as A in his paper. Bond's definition stands squarely upon the experimental facts in so far as they are known, while the spherical albedos previously mentioned depend on more or less serviceable hypotheses which are actually found not to hold with precision.

Besides, one has to deal at times with what Zöllner called true albedo, meaning thereby the specific reflectivity of an element of a diffusely reflecting surface under certain real or supposed conditions, corresponding to some definite case of incidence. This *specific albedo*, as it should be called, is important from the physical standpoint in examining reflecting surfaces where the conditions of incidence are known or can be tentatively assumed, and should properly be confined to normal incidence. Unhappily, Zöllner referred to his spherical albedo, reckoned according to Lambert's law, as "apparent albedo," which has been the cause of much confusion between this quantity and the geometric albedo.

The relation between these three—geometric albedo, spherical albedo, and specific albedo—may be emphasized by the following experiment of the writer, made on a precision photometer using a Lummer-Broadhun contrast prism as indicator. The apparent reflectivity of a ball 16 cm in diameter and painted with several coats of dead-white paint, having a surface as lusterless as could be obtained, showed an absolute value, as measured, of 0.48. The corresponding spherical albedo, assuming Lambert's law, was 0.72. From measurement of the phase factor the spherical albedo, according to Bond's definition (Russell's A), was 0.56, while the specific albedo of the paint used, at small angles of incidence, was 0.74. By this measurement on an artificial planet the three kinds of albedo stand out distinctly in their practical relations. In ordinary photometry the geometric albedo occasionally appears in measure-

[1] *Abhandlungen der K. bayer. Akad. d. Wissenschaften*, **16**, 1888.

[2] *Photometrie der Gestirne*, Leipzig, 1897.

ments of absolute reflecting power, the spherical albedo only in rare instances of curved reflecting surfaces being used as secondary sources of light, while the specific albedo is the quantity which commonly appears in determinations of relative reflectivities which are usually compared at small angles of incidence. As Russell points out (*loc. cit.*), the spherical albedo as defined by Bond has its chief use in considering what happens to the total energy incident upon a planet; Lambert's albedo for all physical purposes is practically useless, as all but a few of the heavenly bodies depart rather widely from it, most of them by unknown amounts. It is unfortunate that it should have obtained so great currency in astronomy, for it has led to many misunderstandings.

CURRENT VALUES OF ALBEDO

The first serious attempt at a comprehensive study of albedo was that due to Zöllner, who was really the founder of celestial photometry. He investigated carefully the light-ratios between Capella and the sun and between the planets and Capella, or some star with which it had been compared, thus finally obtaining the photometric ratios between the planets and the sun, from which he deducted their albedos, using the then current diameters of the planets, which were very imperfectly known. The first investigation included Mars, Jupiter, Saturn, Uranus, and Neptune. Venus and Mercury were added after later photometric measurements.[1] These are all spherical albedos, according to Lambert. In 1893, after most elaborate photometric researches on the planets lasting over some years, Müller determined their relative stellar magnitudes with a precision that, whatever it may really turn out to be, has not since been surpassed. These photometric values he connected with the best measures of diameter then available for a redetermination of the planetary albedos.[2] These albedos were expressed, not as absolute, but as relative values comparable with Zöllner's and assuming Zöllner's value for Mars as unity. These values of Müller's are therefore in terms of Zöllner's sun-Mars light-ratio, this value being probably chosen on account of the fact that from

[1] *Poggendorff's Annalen, Jubelband*, p. 641, 1874.

[2] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **8**, 369.

the large number of observations Mars was supposedly Zöllner's best determined planet. Unfortunately, this has turned out to be his least precise ratio, since, connecting it with Müller's magnitude of Mars, the equivalent stellar magnitude of the sun comes out -26.4. Zöllner's sun-Jupiter ratio gives -26.57, while Saturn, of which Zöllner had a valuable series of observations with the rings gone, would have given -26.9, and Uranus -26.48. Now, assuming the magnitude of the planets to be known, the assumption of too small magnitude for the sun means too high value of albedo all along the line, and when Müller's relative values were translated into absolute values on the basis of Zöllner's Mars, the result was the truly astounding tabulation shown in column 5 of Table I, beginning with Mercury at 0.17 and ending with Saturn and Venus at 0.88 and 0.92 respectively.

TABLE I

ALBEDOS OF THE PLANETS

PLANET	ZÖLLNER		MÜLLER		MÜLLER ⊙ = −26.6
	Relative	Absolute	Relative	Absolute	Absolute
Mercury.....	0.43	0.11	0.64	0.17	0.14
Venus.....	2.33	.62	3.44	.92	.758
Mars.....	1.00	.27	1.00	.27	.22
Jupiter.....	2.34	.63	2.79	.75	.616
Saturn.....	1.87	.50	3.28	.88	.721
Uranus.....	2.40	.64	2.73	.73	.604
Neptune.....	1.74	0.46	2.36	0.63	0.521

The absolute values in this table are all spherical albedos, according to Lambert. These figures have been quoted in most astronomical works from Müller's publication to the present time, in spite of the fact that they were open to suspicion on account of the very high values for Venus and Saturn. The latter, in particular, carried evidence of fatal error on its face, since the assumption of 0.88 for the albedo of the ball as a whole would necessitate considerably higher reflecting power in the bright equatorial belt, and an albedo quite certainly over unity for the bright part of the *B* ring.

Müller himself undoubtedly detected some of the sources of error shortly after his original values were published, and four years later[1] he reverted to Zöllner's sun-Capella ratio which with his own magnitude of Capella led to −26.60 as the stellar magnitude of the sun. This resulted in a general reduction of nearly 20 per cent in the albedos of the planets, which values are set out in the sixth column of Table I. For some inexplicable reason these conscientiously revised albedos attracted comparatively little attention, and the earlier series, like many things that are not so, retained world-wide currency.

ACTUAL STELLAR MAGNITUDE OF THE SUN

In view of Russell's investigation, it is unnecessary to review this matter at length. It should be pointed out, however, that the simple average of the results obtained by Zöllner, C. Fabry, W. H. Pickering, and Ceraski is of dubious value, the chances being that the first and last of these values are entitled to less weight than the others; Zöllner's on account of the widely varying results of his planetary ratios, which indicate large experimental errors, and Ceraski's[2] from the extraordinary variation between his determinations via Polaris, Procyon, and Sirius. When reduced to the Harvard scale from the Potsdam scale, the results from Polaris and Procyon are fairly concordant, but diverge widely as to Sirius, which had fewer observations, but which had the very great advantage of being a much brighter comparison object.

The determination which seems least subject to error from the photometrical standpoint appears to be that of C. Fabry,[3] who obtained −26.80 reduced to the Harvard scale of magnitudes.

Two photometric determinations by Birck[4] and by E. S. King[5] certainly cannot be taken as in any sense decisive, on account of the large and very uncertain corrections for color-index. The same doubt, of course, applies to any other photographic value. In lack of any measurements of the stellar magnitude of the sun

[1] *Photometrie der Gestirne*, Leipzig, 1897, p. 317.

[2] *Astronomische Nachrichten*, **170**, 135, 1905.

[3] *Association française pour l'avancement des sciences*, Part 2, p. 253, 1903.

[4] *Das photographische Helligskeitsverhältnis der Sonne zu Fixsternen*, Gottingen, 1909, p. 74.

[5] *Harvard Annals*, **61**, 56.

which can properly lay claim to high exactness, Russell's tentative value of -26.72 is here assumed as correct, although in the writer's judgment the indications point to a slightly higher value.

It is certain that the quantity needs redetermination badly and by methods which have proved their precision in the ordinary work of photometry. In particular, all color-filters should have their coefficients of transmission determined by the flicker photometer instead of relying on the ordinary instruments in which simultaneous contrast throws luminous values wide from the truth.[1] Secondly, the sector disk, which is by far the most exact and reliable method of cutting down luminous intensities, should be used to the exclusion of every other device for that purpose. By these means and with the powerful electric lamps now available in photometry it should be possible easily to determine the apparent intensity of sunlight within 1 or 2 per cent, leaving the reduction for atmosphere as the only material source of error.

TABLE II

ALBEDOS

Object	m_1	r_1	Geometric Albedo	Lambert's Spherical Albedo	Bond's Albedo Russell's A
Jupiter I	−1.16	2.38	0.46	0.690	0.69 [.48]
Jupiter II	−1.01	2.08	.51	.765	.76 [.53]
Jupiter III	−1.62	3.62	.30	.450	.45 [.31]
Jupiter IV	−0.44	3.49	.11	.165	.16 [.11]
Titan	−1.26	2.9	.33	.495	.50
Ceres	3.70	0.53	.10	.150	.06
Pallas	4.38	0.34	.13	.195	.07
Juno	5.74	0.14	.22	.330	.12
Vesta	3.50	0.27	0.48	0.720	0.26

Given the stellar magnitude of the sun, the two quantities which appear in the determination of geometric albedo are the stellar magnitudes of the objects reduced to unit distance, and their semi-diameters at unit distance. For the small bodies the values of Russell's table, which are in close concordance with the Harvard scale, are adopted, no material discrepancy being introduced by so doing. Table II gives the albedos of the minor bodies thus found.

[1] Bell, "Some Factors in Heterochromatic Photometry," *Electrical World*, January 27, 1912.

ALBEDOS OF THE MINOR PLANETS AND SATELLITES

It is convenient to consider all these small bodies together, since they present in general the same kind of problems, and in particular are all, so far as theory and observation indicate, without atmosphere. Their dimensions preclude the retention of any perceptible amount of atmosphere, the value of gravity at their surfaces being far too small to prevent the escape of even the heaviest gases. If the satellites were derived from their primaries by fission, they would take very little atmosphere with them, and that probably of the lighter constituents. If otherwise acquired, there is still less likelihood of starting with an atmosphere. On any hypothesis, the loss of what atmosphere they may originally have had would be rapid.

The diameters of all these bodies are relatively small, most of them very small, so that measurement of them by ordinary astronomical means becomes exceedingly difficult except in the case of our own moon. All told, and including the moon, only ten of these minor bodies have had their diameters measured with any approach to accuracy. Of the twenty-six known satellites, only five—the original four of Jupiter, and Titan of Saturn's family—aside from our moon, have diameters as yet measured. Of more than eight hundred minor planets but four, constituting the original group, have been measured, and these only by one observer, Barnard.[1] A single, but very good check on the diameter of (4) Vesta, agreeing exactly with Barnard's figures, has been obtained by Hamy, using an interference method. The corresponding geometric albedos are given at the end of Table II. The variations between them are notable, the figures ranging from 0.10 for Ceres to 0.48 for Vesta, bespeaking a very wide difference in surface character; and the recorded observations on these minor planets check well with such variations of albedo, aside from all questions of diameter, Ceres being conspicuously dull and Vesta conspicuously bright. The albedo of the first is well within plausible limits for a dark, rocky material; that of the latter is so high as peremptorily to demand explanation.

[1] *Monthly Notices*, **56**, 55, 1895.

In reducing the geometric to the spherical albedo Russell (*loc. cit.*), has assumed a reduction factor similar to that of our moon; in fact, approximately halfway between the values taken for the moon and for Mercury. This implies, for the asteroids considered, extreme departure from Lambert's law, doubtless due to the strong shadowing, the cause which is effective in the case of the moon and Mercury. The low albedo of Ceres is easily accounted for in this way, but a geometric albedo as high as 0.48 is not consistent with such shadowing as is characteristic of the moon and Mercury, for it would imply a specific reflectivity out of all reasonable range of known massive substances. Zöllner (*Photometrische Untersuchungen*) gives for the moon, based on a study of its slopes and shadowing, a specific reflectivity, "Wahre Albedo" 0.17, while his geometric albedo must be cut down to 0.08. In other words, a body which loses brilliancy from roughness and shadowing, as in the case of the moon, may have a specific reflectivity of surface material something like double the geometric albedo. We must therefore conclude that the brilliancy of Vesta is due both to the texture and to the material of its surface. Pallas and Juno, which have intermediate albedos, may owe them to variations in surface or in shadowing or in both.

The key to this particular problem is found in the phase-factor. A smooth body which approximately obeys Lambert's law will show a phase-factor independent of its surface reflectivity. On the other hand, great roughness, with the accompanying strong shadowing, is likely to produce a large phase-factor, e.g., the moon and Mercury, and to reduce the mean geometric albedo. A rough, heavily shadowed body will therefore show lower albedo than a smooth diffusing one of the same specific reflectivity, and, therefore, other things being equal, *the geometric albedo of an airless body should vary with the phase-factor*, which is an index of the amount of shadowing. The less shadowing, the higher the albedo and the less the phase-factor; the more shadowing, the larger the phase-factor and the smaller the apparent albedo, inasmuch as the effect of shadowing is itself lessened by high surface albedo, a term in A^2 appearing in the reflection formula.

Hence it should be possible to establish an approximate experimental relation between phase-factor and albedo for bodies in which there are no complications due to the presence of an atmosphere. Such is in fact the case. When the phase-factors and geometric albedos of the four well-known asteroids, Mercury, and the moon are plotted in rectangular co-ordinates, as in Fig. 1, they fall into evident relationship. Two additional points, expressing the relation between phase-angle and albedo for a body fairly near to obeying Lambert's law, have been added from the experiments on the artificial planet already referred to, representing two different kinds of diffusing paint, one somewhat glossier than the other. The closeness with which these check the albedo and phase-factor of Vesta serve both to establish that end of the curve and to hint very strongly that the surface of Vesta is both smooth and light in color, as if owing its reflectivity to optical dust. The asteroids vary enormously in phase-factor and hence in geometric albedo. The limits set by the four directly investigated are passed in both directions, the phase-factors varying according to Müller[1] from 0.016 (Iris) to 0.053 (Frigga) magnitude per degree. Even these figures, derived from Müller's list of 34 fairly well-determined asteroids, have been somewhat extended. By graphical interpolation it therefore becomes possible to form an approximate idea of the albedo of an asteroid, provided its phase-factor is fairly well determined, and hence, having the elements of its orbit and its magnitude, to make an estimate of its probable diameter likely to be nearer the truth than any reckoning based on an assumed uniform albedo for the whole body of asteroids, as has been done by previous investigators of this question.

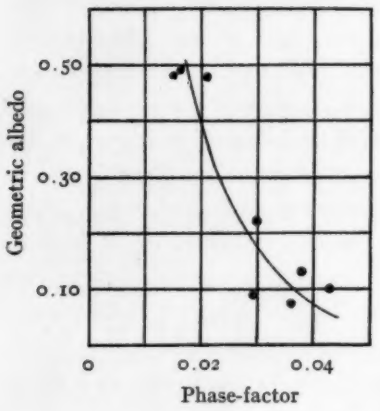

FIG. 1

[1] *Photometrie der Gestirne*, p. 378.

That these albedos are not uniform or anywhere near it, we know to a certainty, and we can establish at least an approximate relation from experimental data between the phase-factor and the probable albedo. Unfortunately, most of the asteroids are not observable through a wide range of phase-angle, and the measurements of different experimenters are in some cases rather discordant. As an example of the application of the relation between phase-factor and albedo to the determination of diameters, the writer selected from Müller's table ten asteroids, of which the phase-factors seemed likely to be the most reliable, considering the number of measurements, the concurrence of measurements by different observers, and the phase-angle covered. The approximate corresponding albedos were estimated from Fig. 1, and the corresponding diameters computed, following the data of Müller's table (*loc. cit.*). The results, including the four experimentally determined asteroids, are shown in Table III.

TABLE III

DIAMETERS AND ALBEDOS OF ASTEROIDS

Asteroid	Diameter at Distance Unity	Geometric Albedo	Phase-Factor	Diameter	Magnitude*
				km	
(1) Ceres	1".06	0.10	0.043	(768)	7.4
(2) Pallas	0.78	.13	.038	(565)	8.0
(3) Juno	0.28	.22	.030	(203)	8.7
(4) Vesta	0.54	.48	.021	(390)	6.5
(7) Iris	0.20	.50	.017	144	8.4
(8) Flora	0.23	.22	.028	168	3.9
(11) Parthenope	0.17	.35	.022	122	9.3
(16) Psyche	0.74	.05	.048	539	9.6
(18) Melpomene	0.31	.14	.033	224	9.3
(30) Urania	0.12	.28	.025	87	9.9
(40) Harmonia	0.11	.50	.017	78	9.2
(44) Nysa	0.17	.28	.025	121	9.8
(77) Frigga	0.31	.05?	.053?	296	11.1
(258) Tyche	0.36	0.05	0.046	259	11.1

* At mean opposition as taken from the *Astronomisches Jahrbuch*, 1916.

A glance at this table shows the great differences which probably exist between the albedos of the asteroids, since at least two of them have albedos as great or greater than that of Vesta and three others are materially less than that of Ceres. The former group, therefore,

showed diameters considerably less than those previously estimated, and the latter group diameters conspicuously greater. (16) Psyche, for example, shows a diameter, both absolute and angular, nearly as great as Pallas, and hence should readily yield to micrometrical measurement, while (77) Frigga and (258) Tyche should be considerably easier to measure than Juno. The data give an interesting clue to the great variation in size of the asteroids. Bauschinger in 1900, on an estimated average albedo of 0.24, found that, of the then known asteroids, (452) Hamiltonia gave a computed diameter of only 10 km. Since his investigation, two considerably less brilliant ones, (719) Albert and 1900 GA, have been added. The latter, estimated to be of the eighteenth magnitude, would, from this figure and its distance, have on Bauschinger's assumption a diameter of only 3.4 km. At the albedo of Vesta this would shrink to less than 2.5 km, while it would be approximately doubled for an albedo like that of Psyche.

It is highly probable, therefore, that asteroids down to less than 5 km in diameter are accessible to photography if of average surface character. On the other hand, a body as dark as Psyche and of this diameter would be too near the optical limit to be detected unless under extraordinarily favorable circumstances, and anything materially smaller would be beyond reasonable chance of observation. As more phase-factors are determined with fair accuracy, asteroids of measurable diameter may be expected to turn up, and the addition of a few more to the four measured by Barnard would give valuable data concerning the physical character of the whole group.

As with increasing numbers they have been followed down to diameters of a few kilometers, there is every probability in favor of Newcomb's belief that they actually grade down to the size of meteorites and cosmic dust. Indeed, if the estimates which have been made regarding the Coon Butte meteorite have sound basis in fact,[1] a celestial body of dimensions fairly comparable with the smallest known of the minor planets actually may have fallen to earth within historic times.

[1] Elihu Thomson, *Proceedings of the American Academy of Arts and Sciences*, 47, 721, 1912.

As the Coon Butte crater is over a kilometer in diameter, the body whose impact caused it could hardly have been smaller than Thomson suggests (about 150 m) and may well have been double this size. As it was a meteorite, chiefly iron, its surface would have very low reflectivity, as low for instance as that of Tyche, than which at the same distance it would have been about 15 magnitudes fainter. At a million miles, however, a body of this size would acquire telescopic magnitude, so that it would comfortably bridge the gap between asteroids and the bodies which, on closer acquaintance, we know as meteors. Our own moon, were it moved to the distance of Ceres, would show a disk a couple of seconds in diameter and would outshine Ceres by more than 3 magnitudes, ranking therefore as the brightest of the minor planets. Incidentally, if the face of the moon we see is a fair sample, the moon would be slightly variable if in regular rotation, in virtue of the effect of the spots, which is found to be in evidence to the extent of nearly 0.2 between the beginning and the end of a lunation.

From the phase-factors, many minor planets have rough and irregular surfaces which bear evidence of an origin not gradual and orderly enough to permit smooth agglomeration or the grading effects of erosion. In this particular, too, they are akin to the meteorites, toward which they seem to pass in size and character. Whatever the origin of meteorites, their composition is such as to indicate little or no free oxygen and water, as was long ago pointed out by Professor H. A. Newton, and, as to the latter disqualification, evidenced by the absence of crystallized silica. There is no clue to the density of the minor planets save such as may be given by that of our own moon and meteorites, the former being about 3.3. The average of the stony meteorites runs a little higher, about 3.5, while the iron meteorites reach about 7.5. The possible asteroids of considerable mass, although of meager dimensions, lend weight to the theory of Seeliger that the aggregate mass of asteroids and of the meteor swarms to which they are allied may be great enough to show some effect in planetary perturbations.

The total light of the whole body of about 800 known asteroids is very small. Even if one took the average as approximately the twelfth magnitude, at unit distance, the whole group would not

equal a fourth-magnitude star. This does not, however, prove that the enormous number of smaller bodies, individually entirely beyond telescopic reach, might not produce a visible effect, particularly if many of them should, like Eros, sweep inward to and beyond the orbit of Mars.

In recent years the average magnitude of the asteroids found steadily decreases. Bearing in mind that it would require the accession of about 10,000 asteroids of the fifteenth magnitude to double the light of those already known, or 1,000,000 of the twentieth magnitude, it is not an unsafe estimate to judge that the aggregate light of the whole swarm can scarcely exceed that of a star of the third magnitude. The question whether this light scattered in an asteroid belt can produce a visible zodiacal belt is easily answered. W. H. Pickering[1] has shown that the brightness of the full moon is equal to that due to 375 tenth-magnitude stars per square second of area, which is approximately 10^9 for the whole surface. Now the known asteroids aggregate little above 100 such stars, and it would be a most liberal reckoning which would give the equivalent of 1000 such stars simultaneously visible in the sky. Even so, the brilliancy of the whole mass concentrated in the moon's area would be barely perceptible, and scattered over the zodiacal belt known to be tenanted by the asteroids the light would be totally beyond view.

A collateral question is the visibility of single scattered bodies which may lie nearer the earth than the asteroid belt as a whole. Suppose a wanderer were to come in to a point only 0.1 unit from our planet, a location almost touched by Eros with its possible 0.12 unit. At the average albedo of the asteroids it would rise, roughly, to something like fifteenth magnitude, even if only a few hundred meters in diameter, and hence could be picked up photographically without much trouble if caught at opposition, only to be swiftly lost in the distance.

Were there many such strays it is safe to say that they would be not uncommon finds by this time; hence one must conclude that there are few brothers to Eros in our neighborhood. Still, a systematic search for small and swiftly moving bodies might well meet

[1] *Harvard Annals*, 61.

reward, considering the very modest dimensions here shown as necessary to visibility. There is some reason to suppose that the best field for such a search might be outside the ordinary zodiacal limits.

RELATION OF ALBEDO TO VARIABILITY

Any notable spotting of a celestial body, so that it presents areas of low and high albedo, will produce variations of light in revolution if the spots are unevenly distributed in longitude, or in orbital position if in latitude, the axis being inclined to the plane of the orbit. If unevenly distributed in both longitude and latitude, the rotational variation may change in magnitude and character with orbital position. Other things being equal, there would seem to be a greater chance of conspicuous spotting in small bodies than in large. Were the moon broken up into asteroids, the chance of getting large local variations of albedo in some of them would obviously be good.

H. N. Russell[1] has investigated the theory of spotted bodies and has shown that for any observed light-variation there may be an indefinitely great number of possible spot-distributions, so that to pass directly from light-curve to a detailed knowledge of the surface is out of the question. This follows, indeed, from the simple consideration that the observed light at any epoch is due to the optical integration of the light received from every element of visible surface taken with its appropriate albedo, $L=\Sigma[a\Delta s+a_1\Delta s'+\ldots.a_n\Delta s_n]$, which expression tells nothing about the positions or albedos of the elements. An aggregation of dark spots interspersed with bright ones would produce exactly the effect of an area of the same mean effective albedo.

But, with this limitation, the light-curve can give some definite clues to surface characteristics. The simplest light-curve would be that of a singly spotted sphere, which corresponds in general to the case of the Algol variables, where the spot is an eclipsing body, with this difference, that the change in light due to a spot would generally be much less abrupt and also smaller. Fig. 2 may be taken as a type of curve from a dull body with a single bright area. The angular

[1] *Astrophysical Journal*, 24, 1, 1906.

extent of the spotting in longitude determines the rate of increase and decrease of the light and the angular extent of the minimum phase. With a spotted area 180° in longitude the curve of variation would cover the entire 360°, and the light-changes would settle into a fairly smooth, regular curve like that shown by ζ Geminorum. If the body shows marked phases, particularly with a large phase-factor, asymmetry is introduced, for then the spot area will pass over only $180° - a$ of visible surface, coming on sharply on the preceding side and passing off slowly on the following side, producing a light-curve of the Cepheid type. Shift of phase-angle may similarly produce slight changes in the magnitude and phase of maxima and minima. Two spotted areas obviously give, if separated considerably in longitude, a double period, which may, from difference of latitude and obliquity of axis, merge into a single period. With spot-areas of different area or intensity and nearly 180° apart one could get a light-curve of the β Lyrae type.

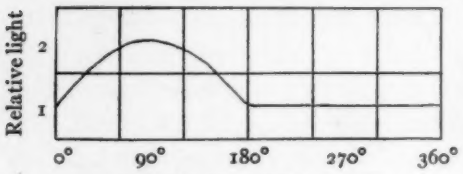

FIG. 2

The quantitative study of the light-curves can add considerably to the inferences drawn from their shape. A body shining by diffusely reflected light, and owing its variations in light to differences of albedo, has a decidedly limited range of light-change. Even were it divided cleanly in longitude into a bright and a dark hemisphere, the one having the albedo of Vesta, the other that of Ceres, the difference of magnitude at 0° inclination of axis would be only 1.7, while with chance spotting, even on a coarse and irregular scale, only small fractions of a magnitude can be expected. Even a body as conspicuously spotted as the moon shows a resulting asymmetry of scarcely $0^{M}.2$ in its light-curve, an amount which long escaped notice. Hence, it is not remarkable that light-changes due to rotation have been rarely detected among the asteroids, generally too faint for easy observation. Entirely fortuitous observational differences of at least $0^{M}.1$ are to be expected in such cases and these are quite sufficient to mask the effect of any moderate spotting.

QUASI-SPECULAR REFLECTION

When light falls upon a plane which possesses wholly or partly a polished surface, the light reflected from it, in part at least, is specularly reflected as from the surface of a mirror. The area considered may be actually a polished plane or may be made up of minute, substantially plane elements presenting slight angles to the general trend of the surface. Every elemental plane which is large compared with the wave-length of light acts as a mirror tending to form a definite image of the source of light. The summation of the rays makes up a brilliant reflected beam of total intensity proportional to the summation of the areas and their coefficients of reflection, and scattered over a solid angle of 2ψ, where ψ is the maximum variation of the individual elements from the normal to the general phase. This is the condition met in a broken surface of crystalline stone which, in the sunlight, will give through a particular, and sometimes quite small, range of angles of incidence an intensely brilliant reflection, dropping to a very much lessened amount after the small angle just indicated has been passed. The familiar reflection of sunlight on the window of a distant house at perhaps many miles away is an example of the same thing, the angle ψ in this case being small compared with the variation of incidence due to the angular semi-diameter of the sun. The distribution of the light thus brilliantly reflected depends on the specific reflectivity of the material, the extent of the source which determines the angles of incidence, and the angle ψ which limits the extent of deviation from the general normal. The smaller and the more accurately plane the reflecting surface as a whole, the less the angle of spread of the resulting beam.

In practice a roughly plane broken area will have a general trend of the reflected ray, a considerable amount of miscellaneous scattering still specular in its character, and a large amount also of diffuse reflection of the ordinary kind due to irregularities of very heterogeneous character as compared with the wave-length of light, but not having their planes so directed as to form a specular beam. The distinction between pure diffusion and purely specular reflection lies chiefly in the magnitude and concordant direction of a considerable portion of the surfaces of incidence. If it is entirely

heterogeneous, the light is diffuse. Possessing general community of direction, it is partly diffuse and scattered and partly of codirected specular effect. And finally, if the surface is nearly plane, the chief element of reflection will be codirected and the effect specular.

Now it is easy to compute the general effect of specular reflection in the case of one of the heavenly bodies. A really plane surface $0''.1$ in diameter would, at distance 2 from the sun and at an equal distance from the earth, reflect, crediting it with a specific reflectivity of 0.2, quite nearly $\frac{1}{5\times 10^{10}}$ the solar light; but the light of the sun equals substantially 5×10^{10} stars of 0 magnitude; hence, if the reflection from this surface were cleanly specular it would shine with something like the brilliancy of Arcturus, and on a shift of phase equal to the solar semi-diameter as seen from it this light would be practically extinguished. A cleanly specular area even $0''.0001$ in diameter would still shine as a star of about the fourteenth magnitude and would be quite obliterated on a similar change of phase-angle. Were the reflecting surface only approximately plane, the resulting beam would be lessened by its spread, but in any case there would be a probability of a very sharp increase in light at one particular angle followed by rapid decrease, leaving, after a swing of a few minutes of arc at the most, only a feeble reflection to take its place. The sharper the specular element of the reflection, and the smaller the element in which it takes place, the more abrupt will be the changes of light ensuing. There is little doubt that we actually get such reflection from some of the wall surfaces of the moon, which contains a very prominent specular element. The variations in brilliancy from point to point as found by W. H. Pickering[1] are so enormous as to offer no other explanation, and the variations of brilliancy with phase of various of these points is so notable as to enforce the same conclusion.

THE LIGHT-ENVELOPE OF A RADIANT

It is customary in photometry to express the intensity of light about a radiant (considered as a luminous point) by a polar curve from this point as origin. For every angle θ reckoned from the

[1] *Selenographical Journal*, 1882.

appropriate axis, a radius r is drawn of length proportional to the total luminous intensity in this direction and its end thus determined defines a point p on the curve. The locus of all points p from $\theta = 0$ to $\theta = 360°$ defines the light-curve in a plane.

Similarly in three dimensions one may express the intensity in any direction in polar co-ordinates, θ, ϕ, for the angles from the two axes of origin, and radii proportional to the corresponding intensities. Then the loci of all points p' determined in varying θ, ϕ,

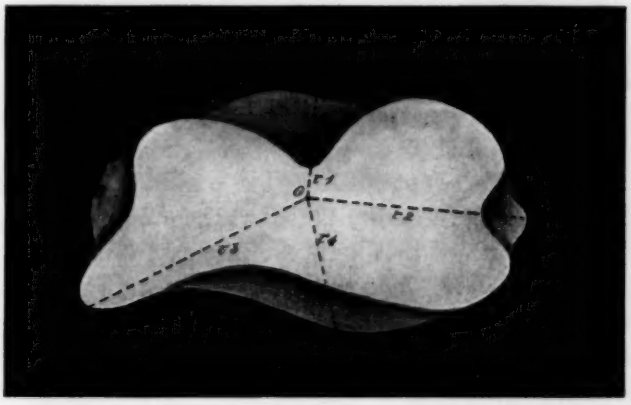

FIG. 3

from 0° to 360° with the corresponding values of r, determine a light-surface of which every radius drawn from the origin represents the total intensity along that angular direction.

In general, this surface is not a figure of rotation about any axis whatever. It must be considered as revolving synchronously with the source if this revolves; while if this be subject to phase-shifting, as in the case of a planet, each shift of phase produces a new set of values for r drawn to the observer and a consequent modification of the light-surface. As a simple example, Fig. 3 shows the light-surface about an electric arc lamp as instantaneous source.

If the source shines by reflected light, a regularly diffusing body will give a relatively symmetrical light-surface, while an area of quasi-specular reflection produces a protuberance due to the increased values of r, of form depending, for any given phase-angle,

upon the area and constants of the specular surface. Except in the case of a perfectly diffusing body of regular shape the form of the light-envelope gives little, and often misleading, information as to the real contour of the body itself.

VARIABILITY OF THE ASTEROIDS AND SATELLITES

Aside from the variations of brilliancy due to phase-angle few periodic changes have been found in the small bodies. In some of the satellites, e.g., the old moons of Jupiter and Titan and Iapetus, there has been found periodic variation of light with orbital position indicating irrotational relation to the primary, as in our moon. Of these, all save the last show variations hardly exceeding half a magnitude, as has been shown by Guthnick.[1] Iapetus presents a different condition, since its range of variation[2] is approximately $1^{M}.4$ (3.6 to 1). This is so great as to be highly improbable as a result of mere spotting. Guthnick[3] shows the range as reaching $1^{M}.8$.

The smooth form of the light-curve found by E. C. Pickering (*loc. cit.*), precludes any great irregularity of form, while the ratio of nearly 4:1 required in the mean albedos of the two hemispheres is too great to be plausible if due to fortuitous spotting alone. A moderate degree of elongation, even less than the 9:11 ratio suggested by E. C. Pickering, with aggregation of material of low albedo toward one hemispheroid, suitably explains the phenomena.

The variability of the asteroids discloses more striking conditions. The phase-factor accounts for most or all of the once surmised long-period variations. Beyond these there appear in a few asteroids well-marked short-period variations clearly rotational in character. Of these, two, (7) Iris, *c*, Fig. 4, and (15) Eunomia, *d*, Fig. 4, investigated by Wendell[4] show variations of small amplitude and simple character taking place in about $3^{h}6^{m}$ and $3^{h}2^{m}$ respectively. Doubtless a great many similar cases will be found when a search is made with instruments of light-gathering power sufficient to make the photometry of these faint objects easy.

[1] *Astronomische Nachrichten*, 198, 230, 1914.

[2] *Harvard Annals*, 11, 67.

[3] *Astronomische Nachrichten*, 198, 252, 1914.

[4] *Harvard Circular*, Nos. 75 and 94.

A radically different type of variability appears in the cases of (433) Eros, and (345) Tercidina, introducing factors of great interest to the photometrist. The first named, carefully studied during the opposition of 1901 by numerous astronomers, disclosed a variation of nearly $2^{M}.0$ with a period of about $0^{d}.11$, or probably double this, if the successive maxima are unsymmetrical. The great variation of light makes simple spotting an improbable cause; the form of the light-curve makes it practically impossible. Moreover, the

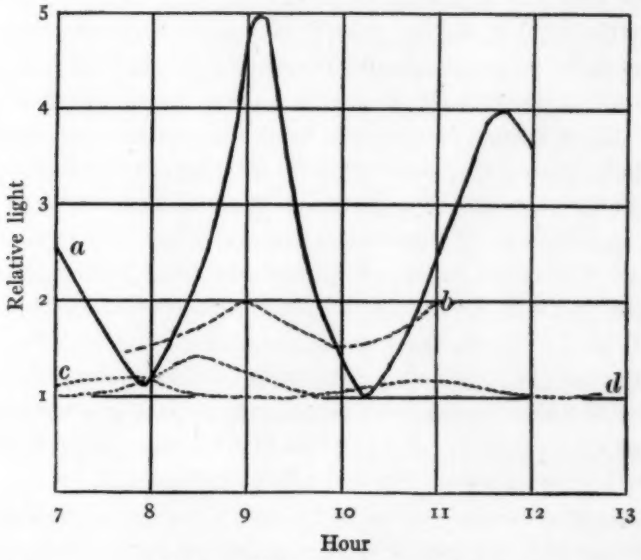

FIG. 4.—Light-ratio curve of Eros, 1901

variation, nearly 2^{M} early in February 1901, had sunk early in April to less than $0^{M}.5$, and in May it became imperceptible.

Fig. 4 shows the actual light-curve of Eros as deduced from the observations of Guillaume, Lecadet, and Lnizet.[1] Curve *a* gives the variations on February 21, 1901, and curve *b* those on April 19 following. Now the amplitudes, and particularly the sharpness of the principal maximum, at once eliminate spotting as a possible cause, since no plausible difference of albedo accounts for the amplitude, with a spot small enough to account for the sharp rise. The

[1] André, *Les Planètes et leur origine*, p. 91.

case is all the worse when the double spotting required to account for the secondary maximum is considered. Likewise eclipsing, as in the Algol class, seems to be barred by the continuous variation and by the same difficulties as to amplitude and rate of variation, taking into account the limited range of albedo to be expected. E. C. Pickering[1] has gone over these hypotheses and has very properly rejected them. The kindred hypothesis of greatly elongated form requires a high average light at times of little or no variability, contrary to the observed facts.

The curve *b* adds to the complications, for it shows complete regularity within the experimental errors combined with a very rapid change in amplitude for a small change in relative positions of the earth and Eros. In case of eclipsing bodies moving about an inclined axis, failure of eclipse through shift of aspect should generally raise the average light instead of decreasing it.

Curve *a*, however, though extraordinary from the astronomical standpoint, concerned chiefly with self-luminous bodies, is of a form perfectly well known in technical photometry. It is an entirely typical case of quasi-specular reflection, and not a very extreme one at that. Curves at varying incidence from surfaces showing mixed specular and diffuse reflection always present these phenomena of sharp rise, a maximum which narrows with the angle subtended by the specular area and a fall which may or may not be symmetrical with the rise, according to the structure of the surface. An admixture of a few per cent of specular in the general diffuse reflection would give a curve like *a*.

The light-surface resulting from such a surface takes the form of a protruding horn of moderate angular extent, the point of which may be shifted out of the line of sight by a very small apparent angular displacement of the axis of revolution due to change of orbital position. With increasing displacement the reflection becomes in effect less and less specular, the amplitude of light-change greatly decreases, and may quite disappear, together wth any asymmetry due to the specular surface.

From this viewpoint we need consider Eros only as a somewhat irregularly flattened mass revolving on an inclined axis lying

[1] *Harvard Circular*, No. 58.

between the flattened surfaces. At a single point during opposition the maxima of its light-surfaces may sweep into the line of sight and then pass away from it rather rapidly, leaving only the trivial variations due to a diffusing body somewhat irregular in form and texture. The great eccentricity of the orbit of Eros and its inclination of 10°50′ produce so great variation in the geometric conditions of successive oppositions that many years may elapse before the photometric variations of 1901 are reproduced. It is not surprising, therefore, to find that at the prior dates cited by E. C. Pickering (*loc. cit.*) and at the opposition of 1907, carefully observed by Guthnick,[1] varying results are shown, in the latter case no definite short-period variation, save a suspected one of $0^{M}.25$, with double maxima and minima in a scant 5 hours, derived from the observations in the middle third of October. Guthnick did find, however, a much smaller phase-coefficient and also an apparent magnitude less by at least $0^{M}.4$ than were found in 1901, which is quite what one would expect on the present theory.

The Harvard observations of 1896[2] are in fair agreement with the figures obtained two oppositions later.

At the opposition of 1903 a long series of measurements of Eros was obtained by Bailey at Arequipa, together with others on several asteroids.[3] These showed a double period of $0^{d}.2196$ for Eros, with an apparent range of $0^{M}.5$ to $0^{M}.75$. The photometric and photographic results agreed closely. The periods deduced from the several groups of observations concurred well, but the curves show strong indications of some variation in amplitude and perhaps in form, although the general type was sinusoidal, as in the later stages of the previous opposition. The time of revolution found is substantially identical with that already noted during the preceding opposition after the amplitude had fallen into a regular sinusoid.

Two oppositions later Guthnick found, as already cited, no variation save during a brief season, when the period appeared as somewhere near $0^{d}.2$, as before. During the opposition of 1914 Miss Margaret Harwood[4] investigated the variations photographically and found an amplitude of approximately $0^{M}.5$, but with the

[1] *Astronomische Nachrichten*, **178**, 1, 1908.
[2] *Harvard Circular*, No. 58.
[3] *Harvard Annals*, **72**, 165.
[4] *Ibid.*, **76**, 161.

startling period of $0^d.3064$, some 2^h5^m longer than before. It seems clear from her data that the former period could not possibly meet her observations, while the longer one did. Incidentally her light-curve shows departure from the previous forms, in having a relatively short minimum.

The facts, however, do not demand a real change of rotation period on the part of Eros, the inherent improbability of which is enforced by the uniform regularity found up to the opposition of 1907. Any cause ultimately producing so great a change can hardly be expected to be inactive for so long and then suddenly potent without any observed effect on the orbital motion of the planet. Considering the photometric history of Eros an explanation at once suggests itself. If the light-envelope of a body in any aspect is approximately a surface of revolution, the visible intensity is substantially constant so far as rotation is concerned. If this surface is other than a spherical zone, by change of angle between the axis of revolution and the line of sight, the radius of the envelope will change from r to r' and the light will vary in the ratio $\frac{r}{r'}$. Hence any periodic shift of axis will produce variability not dependent on the period of rotation and generally non-sinusoidal.

Now, Eros, from its erratic light-curves, appears to be a sort of Maxwell's top, spinning in space, and subject to the characteristic wabbling precession of an unbalanced rotation, unchecked by the internal viscosity that acts as stabilizer in the major planets. From this viewpoint, the apparently lengthened period of 1914 is not a rotation period at all, but the effect of a sort of nutation period, or subperiod, on the light-envelope, resulting in the somewhat unsymmetrical period shown. The effect of such a motion would be to impress irregularities on ordinary rotational light-changes in certain positions of the axis with reference to the line of sight, and such seem to be in evidence in the observations of 1901 and 1903. The light-changes of Eros are quite inexplicable as due to spotting, but are what may reasonably be expected of a somewhat irregularly flattened body revolving about an axis considerably inclined to the plane of its orbit, with the rapid and wabbling precession due to the relations between the axes of its ellipsoid of inertia. In such case

the light-variations in the next few oppositions may follow those of the last, recur to former habits, or start on a fresh line, since the gravitational forces cannot be lost sight of at opposition. The condition of staggering in rotation was foreseen by Seeliger,[1] and it now seems probable that the corresponding light-variation has been observed.

(345) Tercidina seems to present conditions similar to those of Eros in changes of variability, probably owing to a like cause, though the apparent staggering in path shown on one of Wolf's plates[2] can hardly be a genuine phenomenon. Its only dynamical analogue is the spiraling flight sometimes found in rifled projectiles, but due to the resisting medium, inapplicable to the astronomical case.

One other instance, that of (77) Frigga, which became entirely invisible for six consecutive oppositions, may fall into the same category. It has a very large variation with phase-angle, showing marked irregularity of some sort, and if this is associated with a large proportion of specular reflection, it is by no means unlikely that precessional staggering may from time to time reduce its light enough to make it difficult of observation. It certainly deserves to be watched. In fact, any asteroid with a large phase-angle is worth examination for variability. The short periods indicated in the few cases investigated, and the appearance of new types of variability, give hope of disclosures of importance regarding the origin of the group. One must, however, point out that the irregular form of the light-envelope, which bears no definite relation to the form of the generating body, makes it necessary to exercise extreme caution in drawing dynamical conclusions from the photometric facts.

SOME PECULIARITIES OF THE SATELLITES

The common variability of satellites with orbital position, beginning with Cassini's study of Iapetus, is familiar. It holds true for the four old satellites of Jupiter, for Titan, and, with two or three possible exceptions, for the rest of Saturn's tribe. Of this

[1] *Astronomische Nachrichten*, 155, 78, 1901.

[2] *Ibid.*, 156, 87, 1901.

list Titan presents no remarkable features save the high geometric albedo of 0.33. It has, approximately, the diameter of our moon, and the same density, near to that of a stony meteorite, and about the same degree of variability as that indicated for our lunar surface, suggesting miscellaneous spotting on a scale telescopically irresolvable, as would probably be the case were the moon at a similar distance. But its albedo is nearly four times as great as the moon's. The difference is rather too large easily to account for in bodies of similar constitution, and the natural inference is that Titan suffers less from shadowing and very likely carries considerable optical dust, which is much in evidence in Saturn's neighborhood, perhaps in part owing to impact with meteorites known to have raided Saturn's system on one,[1] and probably on many, occasions. A beaten surface of rock may rise to Titan's albedo, while a solid surface is extremely unlikely to rise so high, judging from terrestrial materials.

The four old satellites of Jupiter present an entirely different case. They are all characterized by very low density: I, not much in excess of that of water; II, barely twice that figure, and the others between. All have at times been noticed as bearing visible shaded areas, clear variations from circular contour have been noted,[2] and all exhibit light-variations of an interesting character—regular with orbital position, apparently permanent, and surprisingly sudden, especially in the cases of I and II, which also have the highest albedos. The curves given by Guthnick (*loc. cit.*) for III and IV, while varying systematically, are smoother and cover less range. The first obvious deduction is that one must put aside W. H. Pickering's interesting hypothesis, to account for low density and apparently variable markings, that these bodies are loose aggregations of matter. Aside from the necessary stringing out of such a mass by collisions, the persistence of sharp changes of brilliancy in the same reasonably restricted longitudes for more than a quarter-century clearly marks the satellites as definite solids. Especially in II, the very abrupt light-changes are characteristic of the shifting of a region of quasi-specular reflection. No mere spot of different albedo can well shift the total light back and forth

[1] *Monthly Notices*, 74, 101, 1913. [2] *Annals of the Lowell Observatory*, 2, 30.

by 20 per cent in as many degrees of longitude. Satellites III and IV, too, show sudden, though less marked, peaks of brilliancy, and it is not impossible that Barnard[1] may have actually glimpsed one of the specular regions during a transit of III.

The abrupt light-changes, too, make it clear that these satellites cannot at all follow Lambert's law, and consequently Russell's values for A, quoted in Table II, must be radically revised. Assuming Russell's reduction factor q as for our moon, one reaches the values in brackets. From the high geometrical albedos of I and II a very high reflectivity would follow if one were dealing with pure diffuse reflection, hardly to be accounted for save by the presence of light optical dust as in the case of Titan, but there must be at least small quasi-specular effects from the shape of the light-curve.

This shape also makes it clear that these satellites can have no great amount of atmosphere, of which the diffusion in transmission and reflection must inevitably produce fairly regular diffusion. The low densities point to constitution very different from our moon, to material, in fact, of exceptionally low specific gravity as a mass. Perhaps pumice comes as near to the physical requirements as any terrestrial material, having a specific gravity of from 0.7 upward, and specific reflectivity from 0.25 when dense, up to at least 0.70 when reduced to optical dust, with frequent examples of quasi-specular effects.

Concerning the outer satellites of Jupiter and Saturn and the few others of the solar system it can only be said that most give indications of variable light with orbital position, but that of their albedos there is no definite indication, even the phase-angle relation being denied us. Hence estimates of diameter from the light reflected are futile, save as the limits of albedo confine them. As between the highest and lowest albedo of the small bodies there is a ratio of about 9:1; hence it may fairly be assumed that a diameter may be certain to a 3:1 ratio, or if the assumed albedo be taken at a mean ratio for the geometric albedos ascertained for the small bodies (very nearly 0.20), then the diameter of the unknown is quite certainly determined within 1.5:1, and probably still closer.

[1] *Astronomische Nachrichten*, 144, 325, 1897; and quoted in André's *Les Planètes et leur origine*, p. 192.

SUMMARY

The subject of albedo, forming an important clue to the physical characteristics of the heavenly bodies, has been greatly befogged by misunderstanding as to the definition of the term. Three quantities have been so called: (1) the geometric albedo, which is merely the apparent reflectivity of the body at full phase, as determined from its photometric magnitude and position in space; (2) the spherical albedo, in which the shape of the reflecting body and the relation of incident and reflected rays, according to some known or assumed laws, are taken into account; the commonest form of this assumes the application of Lambert's law to a perfectly diffusing sphere, which gives a numerical value 3/2 the geometric albedo of the same body, and equal to the specific reflectivity of an element of surface at normal incidence; (3) the specific reflectivity of an element as found or deduced from a definite assumption as to incidence, usually taken as normal.

Albedo in astronomical literature has most often meant Lambert's albedo. The commonly current values of this quantity due to Müller[1] are not only much in error, but, in spite of the fact that Müller himself partially corrected them twenty years ago, have often led to fantastic efforts to explain, in defiance of physical facts, figures without substantial basis.

The chief sources of error were a value too low for the stellar magnitude of the sun and some loose data regarding planetary diameters. These have been corrected by Russell, whose values for the geometric albedo p have been assumed in this investigation with the mental reservation that they may be a slight percentage high. Russell's A is spherical albedo on Bond's definition and is not here employed.

There exists among the small bodies having no atmosphere a relation between the variation of light with phase-angle and the geometric albedo, both in substance and in theory, owing to the fact that both quantities depend in considerable measure on the heavy shadowing of a rough surface. This relation leads to a graphical method of estimating albedo from phase-factor, serviceable in approximating the diameters of small bodies. In default of knowl-

[1] *Publikationen*, Potsdam, 8.

edge of phase-factor a geometric albedo of 0.2 is likely to give the minimum percentage of error in diameter.

From these data the asteroids grade down steadily to diameters of a few kilometers and probably to meteoritic dimensions. Their total luminosity is, however, insufficient to account for visible glow, which must be attributed to matter lying within the asteroid belt.

The variability of the small bodies cannot satisfactorily be accounted for by local differences of albedo, since the range is often too great, that of so conspicuously spotted a body as our moon being hardly $0^{M}.2$, and in some cases (e.g., Eros) the form of the light-curve is typically that due to another and well-known cause. This is quasi-specular or "mixed" reflection due to a specular component, as from definitely aligned faces, superimposed on diffuse reflection. It produces more or less definitely directed beams of relatively high intensity, and gives the reflecting body a light-envelope of very unsymmetrical form. The variations in the light of Eros at its various oppositions are such as would be produced by a somewhat flattened body revolving on an axis considerably inclined to the orbital plane and passing between the flattened areas, which show specular components. Change of angle between axis and line of sight leads to small and regular variations like those late in 1901, which may finally vanish. The apparent change of period in 1914 can hardly be due to real slowing of the rotation, considering the steadiness from 1896 to 1907, but may readily be accounted for by the staggering precessional motion of an irregular body, a cause of variation suggested by Seeliger in 1901. Tercidina and Frigga furnish possible cases of the same sort.

The satellites, like the asteroids, give large variations of apparent albedo, which show in the different aspects presented around their orbits. These may be complicated by elongated form, but are equally well accounted for by slight specular factors, plainly in evidence in Jupiter's I and II. These light-curves confirm the lack of atmosphere, and their permanence shows the Jovian moons to be solid in spite of their low density.

BOSTON, MASS.
November 1916

A QUALITATIVE DETERMINATION OF THE REFLECTION COEFFICIENTS OF SOME METALS IN THE SCHUMANN REGION

By I. C. GARDNER

The great majority of the investigations of the optical properties of metals have stopped short at λ 1850. This wave-length constitutes a natural boundary in the spectrum, as the absorption of air makes necessary, for work with light of shorter wave-length, the use of a vacuum spectroscope and Schumann plates with accompanying complications. If we proceed farther into the region of extremely short wave-lengths, we soon reach a point (λ 1250) beyond which light is transmitted by no solid substance, so far as is known. We are therefore soon forced to use in any optical system, not refracting, but reflecting members exclusively. It is then evident that a knowledge of the reflection coefficients of various substances is necessary, before apparatus is designed for work in the region of extremely short wave-lengths.

The work which has been done upon the reflection coefficients of metals for light of wave-lengths lying between, say, λ 2800 and λ 1850 may be briefly reviewed. Schumann,[1] in collaboration with L. Mach, published an account of an investigation he made on alloys in different proportions of aluminium and magnesium. The work is entirely qualitative and extends as far as λ 1850. Two alloys, No. IV (Al 1.25, Mg 1) and No. V (Al 27, Mg 24.3), reflected the ultra-violet better than anything else he tried. Hagen and Rubens[2] have done a great deal of work upon metallic reflection, but they did not include the region shorter than λ 2500. They studied eleven metals and with one exception found that they were characterized by low reflection coefficients (less than 50 per cent) for λ 2500. At this point they found magnalium to have a reflection coefficient of approximately 67 per cent. Glatzel[3] measured

[1] *Sitzungsberichte der K. Akad. der Wiss. in Wien*, **108**, IIa, 154, 1899.

[2] *Zeitschrift für Instrumentenkunde*, **22**, 42, 1902.

[3] *Physikalische Zeitschrift*, **2**, 176, 1900.

the reflecting power of silver, steel, nickel, and copper as compared with Brandes-Schünemann alloy. He does not state explicitly the shortest wave-length with which he worked, but it seems probable that it was about λ 2380. Beyond λ 2500 none of the metals tried was superior to the alloy. Nutting[1] investigated the reflection coefficients of ten metals by a photographic method and gives graphs showing variation of reflection coefficient for the region extending from λ 4000 to λ 1850. At λ 1850 he ascribes to aluminium and brass a reflection coefficient of 65 per cent. The other metals fall below this, with 35 per cent for speculum metal. At approximately λ 2200 the absorption of the gelatine of the photographic plate begins to play an important rôle, and it seems improbable that quantitative results of much accuracy can be obtained beyond this point without the use of Schumann plates.

In all of the work described above the reflection coefficient has been determined by the "direct" method. There is a second method, the "indirect," in which the reflection coefficient is derived from a measurement of the constants of the elliptical polarization produced when a beam of plane-polarized light is reflected from the substance in question. A very convenient method of doing this has been devised by Voigt[2] and applied by Minor,[3] Meier,[4] Fréedericksz,[5] Erochin,[6] and Foersterling and Fréedericksz.[7] These five papers provide data obtained by the same method for some twenty different metals extending in some few cases as far as λ 2260 and in nearly all cases to λ 2573. In general, the reflection coefficient for the shortest wave-length measured lies between 20 and 40 per cent. The following values are perhaps worthy of mention: silver, λ 2263, 18.4 per cent; platinum, λ 2573, 37.1 per cent; mercury, λ 2570, 57.9 per cent; Wood's metal (composition not given), λ 2573, 52.7 per cent; chromium, λ 2570, 69.8 per cent; and tin, λ 2570, 71.4 per cent.

The most recent and by far the most extensive work upon the reflection coefficients of metals which extends to λ 1850 has been

[1] *Physical Review*, **13**, 193, 1901.

[2] *Physikalische Zeitschrift*, **2**, 303, 1901.

[3] *Annalen der Physik*, **10**, 581, 1903.

[4] *Ibid.*, **31**, 1017, 1910.

[5] *Ibid.*, **34**, 780, 1911.

[6] *Ibid.*, **39**, 213, 1912.

[7] *Ibid.*, **43**, 1227, 1914.

carried on by Hulburt.[1] He used the direct method and measured the intensity of light by means of a photo-electric cell. Twenty-eight metallic mirrors were tried, namely aluminium, antimony, bismuth, cadmium, carbon, carborundum, chromium, cobalt, copper, gold, lead, magnalium, magnesium, molybdenum, nickel, palladium, platinum, selenium, silicon, silver, speculum metal, steel, stellite, tantalum, tellurium, tin, tungsten, and zinc. For each of the metals tried there is published a graph showing reflection coefficient as a function of wave-length between λ 1850 and λ 3800. With one exception, carborundum, the trend of the curve as we pass to the region of shorter wave-length is downward. In the case of carborundum the reflection coefficient is 15 per cent at λ 3800 and rises to 20 per cent at λ 1850. Furthermore, with one exception, silicon, 60 per cent, the reflection coefficient is 30 per cent or less at λ 1850. Magnalium, for which, in view of the work of Schumann and Mach, and of Hagen and Rubens, referred to above, we should expect a large reflection coefficient, reflects less than 10 per cent of the light at λ 1850.

There are no data dealing with the optical properties of metals for wave-lengths less than λ 1850. Lyman[2] has recently extended the spectrum to λ 600, and it therefore seems desirable to continue the investigation of reflection coefficients into the region beyond λ 1850. The minimum wave-length dealt with in this paper is λ 1030, at which point we come to a second natural boundary in the spectrum. By the use of extremely pure hydrogen and an oscillatory discharge, the spectrum may be extended to λ 900, and the extension to λ 600 was achieved by substituting an atmosphere of helium for hydrogen in the spectroscope.

Since the difficulties of measurement of energy in the Schumann region have proved prohibitive up to the present time, and since the nature of the Schumann plate renders the results of photographic photometry of very questionable value, it has not seemed advisable, in the present work, to attempt to obtain quantitative determinations of the reflection coefficients of various metals such as is desirable for the testing of theoretical treatments of the optical

[1] *Astrophysical Journal*, 42, 205, 1915.

[2] *Ibid.*, 43, 89, 1916.

properties of substances. The purpose has therefore been restricted to the investigation, in a qualitative manner, of the reflection coefficients of several metals in order to determine what ones are best suited for use in the construction of apparatus for work in the region of extremely short wave-lengths.

As speculum metal has in the past been used for the construction of gratings for the resolution of Schumann light, it was adopted arbitrarily as the standard reflecting metal with which the other metals were compared. We obtain then, not the reflection coeffi-

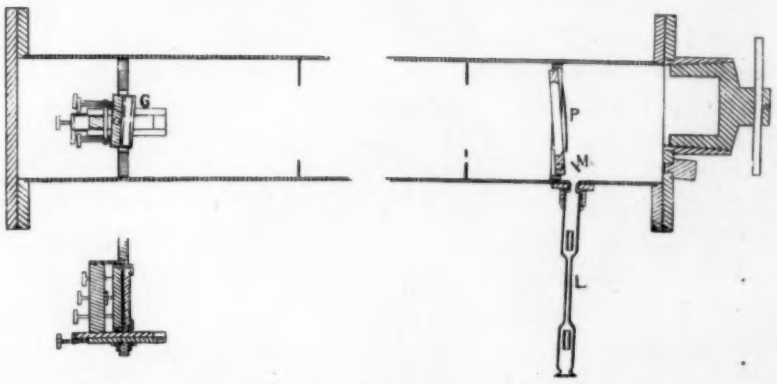

FIG. 1

cient directly, but determine at once what metals are superior to speculum metal for the making of gratings and other optical parts.

A bit of speculum metal and the metal to be tested were so mounted that their polished faces were adjacent and in the same plane. They were then placed in such a position that the light from the quartz discharge tube was reflected (angle of incidence approximately 45°) through the slit and upon the grating. One half of the slit was illuminated by light reflected from speculum metal, the other by light reflected from the metal to be tested. For light of the wave-length to be studied, the astigmatism of the grating in the first-order spectrum is so slight that we obtain simultaneously two spectra side by side, the relative strengths of which enable us to determine by inspection which is the better reflector. The method is illustrated in detail in Fig. 1, which is a horizontal section taken through the vacuum spectroscope and discharge tube. *L* is

the discharge tube which serves as source of light, M is the composite mirror, and G is the grating. The spectrum is received on the photographic plate P. The grating used has a radius of curvature of 102 cm, and is ruled with approximately 6000 lines to the centimeter. The ruled portion is 2.8×5.3 cm. By photographing the two spectra at the same time we eliminate any effect due to variation of intensity of the source. There remains an outstanding error due to possible unsymmetrical placing of the source in such a manner that the two mirrors are unequally illuminated. This may be largely eliminated by interchanging the two mirrors and taking a second exposure. This method, while simple, is sufficiently accurate for the purpose of the work. For if the reflecting powers of two metals are so nearly identical that the eye detects no differences in the two images on the Schumann plate, obviously one metal is, so far as the reflection coefficient is concerned, as suitable for use in the construction of apparatus as the other.

The vacuum grating spectroscope used was patterned very closely in its essentials after the one used by Lyman.[1] It differed in that it used a much larger plate (2.5×8.5 cm) which in turn made necessary the very large hole for the removal of the plate-holder. This hole was closed by a conical plug 9.6 cm in diameter. The reflection of the light by the mirror necessitates the placing of the discharge tube on the side of the spectroscope instead of on the end plate. A novel feature, perhaps worthy of mention, is the mounting of a miniature incandescent lamp on an arm so that it may be swung down immediately in front of the center of the grating. If the mirrors are in correct adjustment the light from this lamp passes through the slit of the spectroscope, is reflected by the mirrors down the capillary of the discharge tube, and the image of the slit may be seen by looking through a telescope pointed along the axis of the capillary. It would have been extremely difficult to test the adjustment of the mirrors without some such arrangement.

The quartz discharge tube which served as source of light was of the external capillary type. It is shown in section at L, Fig. 1. The electrodes were of aluminium, and the internal diameter of the

[1] *The Spectroscopy of the Extreme Ultra-Violet*, Longmans, Green, & Co., 1914, p. 34.

capillary was about 0.6 cm. There is no window at the inner end separating the interior of the discharge tube from the interior of the spectroscope. The spectroscope is commonly filled with hydrogen at a pressure of approximately 1.8 mm and the spectrum obtained is that of hydrogen. The spectrum usually extends from λ 1640 to λ 1030 and is filled with lines very close together.

The directions for making plates given by Schumann[1] were followed quite closely. A device for dropping the plate-holder controlled by a magnet on the outside and similar to that used by Lyman[2] was provided so that four exposures might be taken on one plate without opening the spectroscope. The lengths of the four exposures were usually 30, 60, 120, and 240 seconds.

The method has been applied to nine substances, namely, silicon, gold, stellite, platinum, copper, nickel, aluminium, silver, and steel. The majority of the metals were selected because the ease of working and permanence of surface seem to make them adaptable for the construction of mirrors, if they should prove to be good reflectors. Silicon was chosen as a direct result of the work of Hulburt, who attributed to it a reflection coefficient of 60 per cent at λ 1850. The aluminium was very difficult to polish. I was led to test it by the fact that Nutting (*loc. cit.*) gives it a reflection coefficient of 65 per cent at λ 1850. Following will be found a complete account of the preparation of the mirrors and the results obtained with them.

The speculum metal mirrors used in this work were sawed from a broken grating and polished by Alvan Clark & Sons, Corporation. They were repolished from time to time as conditions demanded.

Silicon.—The specimen of silicon was obtained from the Carborundum Works at Niagara Falls and was sawed into slabs and polished by means of rouge on a beeswax lap at the laboratory. Silicon polishes easily and the surface is permanent and does not tarnish. Silicon is much the best reflector of all the substances that I have tried. Throughout the entire range (λ 1640–λ 1030) it is a better reflector than speculum metal, although the difference is less marked for wave-lengths shorter than λ 1300. Between λ 1600

[1] *Annalen der Physik*, 5, 349, 1901. [2] *Op. cit.*, p. 35.

and λ 1300 I estimate that the reflection coefficient of silicon is twice as large as that of speculum metal. This estimate is onl approximate and indicates that, to the eye, the spectrum reflected from silicon with a two-minute exposure is fully as strong as that reflected from speculum metal with a four-minute exposure.

Gold.—The gold mirror was prepared by deposition from a cathode. Gold spatters very readily, and from six to ten minutes are sufficient to produce an opaque film. The spectra reflected from gold and from the speculum metal are very nearly identical in strength. Between λ 1300 and λ 1030 gold is perhaps slightly superior, but the difference is not sufficiently marked to encourage the use of gold for mirrors.

Stellite.—The stellite mirrors were cut from a block of stellite obtained from the Stellite Works, Kokomo, Indiana. It is an alloy, chiefly of chromium and cobalt. The exact composition is a commercial secret. The mirrors were polished by means of diamantine on a beeswax lap. Hulburt found stellite superior to speculum metal between λ 2000 and λ 3000. His diagram indicates that its reflecting power falls off as we approach λ 1850. The spectra reflected from the speculum metal and stellite are identical in strength. In the Schumann region there is no advantage in substituting stellite for speculum metal.

Platinum.—The platinum mirrors were deposited on glass from a cathode. About thirty minutes were required to deposit an opaque film. Between λ 1600 and λ 1300 platinum is not superior to speculum metal, in fact one or two of my negatives indicate that it is slightly inferior. But beyond λ 1300 the spectrum reflected from the platinum is distinctly the stronger. It would be very interesting to investigate the reflecting power of platinum throughout the region discovered by Lyman (λ 1250–λ 600). As noted above, silicon is a much better reflector than speculum metal in the portion of the spectrum with which this article deals, but, as compared with speculum metal, its reflecting power is growing less as we pass to the shorter wave-lengths, whereas in the case of platinum the reflecting power is increasing.

Copper.—Copper spattered readily on glass from a cathode. The mirrors tested were quite opaque. The copper mirrors which

I made were markedly inferior to those of speculum metal through the entire region. Hulburt finds that at λ 1850 the copper was slightly better as a reflector than the best speculum metal mirror which he tested.

Nickel.—About three hours were required to produce a brilliant opaque film on glass by cathode deposition. Between λ 1600 and λ 1300 the reflection from nickel and from speculum metal is the same. Beyond λ 1300 the nickel is very slightly superior to the speculum metal.

Aluminium.—Two pieces of aluminium cut from commercial sheet stock were polished by Alvan Clark & Sons, Corporation. It was very difficult to polish, but a surface was finally obtained which was quite good. Aluminium reflects very poorly in the Schumann region. Only a few of the stronger groups of lines show in the four-minute exposure.

Silver.—The silver mirror was cut from a coin and polished. Speculum metal is much superior to silver over the whole range upon the plate. At the end of longer wave-length the speculum metal reflects twice as well as does the silver.

Steel.—The steel mirror was hardened and polished by Alvan Clark & Sons, Corporation. Steel reflects markedly better than speculum metal from λ 1600 to approximately λ 1300. At λ 1200 the speculum metal reflects slightly better than the steel.

Conclusion.—The reflecting power of nine metals, silicon, stellite, copper, nickel, gold, platinum, aluminium, silver, and steel, has been investigated in the portion of the spectrum lying between λ 1600 and λ 1030. The purpose has not been to determine the reflection coefficients on an absolute scale, but rather to determine directly what metals are superior to speculum metal for the construction of reflecting surfaces to be used for light in this region. The results indicate that:

1. Silicon is much the best reflector of all the metals tried, and over a portion of the region it reflects the light approximately twice as well as speculum metal.

2. The reflecting power of platinum as compared with that of speculum metal is increasing as we pass to shorter wave-lengths.

It may possess advantages over other metals for work beyond λ 1030.

3. Stellite, copper, nickel, gold, and steel show only minor variations of reflection when compared with speculum metal.

4. Silver and aluminium are very much poorer than speculum metal, with silver ranking above the aluminium.

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November 1916

THE UNITS AND NOMENCLATURE OF RADIATION AND ILLUMINATION

By HERBERT E. IVES

Illumination and its measurements (photometry) are, strictly speaking, merely departments or special cases of radiation and radiometry. In a comprehensive treatise on radiation, photometry could be discussed as such a special case with considerable resultant economy both of space and of the student's mental effort. Thus the various quantities dealt with in radiation become parallel illuminational quantities when passed through a template or filter yielding a spectral luminosity-curve. The operation of measureing light with a photometer is merely an example of the use of a null method of radiometry with the eye as the sensitive radiometer. The greater part of practical photometry, i.e., photometry with no difference of color, could thus be dismissed. The complications introduced when the radiations are of different composition then take their place as disturbances due to the inability of the eye simply and definitely to evaluate as light radiations of various wavelengths. Unlike other selective radiometers, which are selective merely in responding in different *degree* to various radiations of the same intensity, the eye recognizes or creates a difference of *quality* as well. The problem of photometry with a difference of color is that of eliminating the confusing effect of this difference of quality.

However, if the attempt is made to handle photometry in this way, a difficulty is met at the outset in the fact that the derived science, photometry, has been worked out in much greater detail as to instruments, methods, and terminology than has the parent science. On going back from things photometric to things radiometric many of the latter are found to be missing or else in confusion. This remark applies with particular emphasis to the terms used to describe the various radiational quantities. The nomenclature of illumination, on the other hand, has been put into businesslike shape by technical usage and by painstaking committees of the

technical societies, with a resultant economy of words and clearness of meaning that are sadly lacking in the more fundamental science.

To indicate briefly the existing confusion, which later will be dwelt on more in detail, it may be said that the word "radiation" is used as the title, not only for a branch of physical science, but for the "stuff" radiated; for the process of ejecting energy from a body without regard to where it goes; for the process of throwing the energy upon a body without regarding whence it comes; and for either the quantity of energy or the rate of transfer of energy. When this stage is passed and an attempt is made to differentiate between various manifestations of radiant energy each author seems to make it a point of honor to use such terms as "emission," "emissive power," and "intensity" in a sense quite different from anyone else. I have yet to find an author who has conscientiously selected and arranged in tabular form a set of terms for radiational quantities in a manner comparable to that found quite indispensable when a branch of science reaches the stage of application.

A very considerable part of the problem of treating illumination as a special case of radiation therefore appears to consist in the standardization of the terminology of radiation, and this note is concerned largely with suggestions to that end.

The accompanying table (I) of radiational and illuminational quantities is an attempt to arrange these in parallel columns in the manner suggested at the outset. Logically, the illumination columns should have been derived from those of radiation. Actually, the greater completeness and orderliness of that technically applied division of radiation fits it to act as a guide to the standardization of the terminology of the more fundamental quantities. The table thus consists of practically the accepted illuminational quantities with a set of suggested terms for the radiational quantities, the latter culled from a variety of sources and chosen with regard to freedom from ambiguity and to similarity to the parallel illuminational quantities. Justification for the selection of names will be ventured below.

In the first column are the radiational quantities; in the second are these same quantities after the radiation has been passed

TABLE I

Relations between Radiation and Illumination

Radiation		Illumination		
	After passage through a luminosity-curve template become		After division by the mechanical equivalent of light become	
1. Radiant energy (joules)		Luminous energy (light-watt seconds)		Luminous energy (lumen seconds)
2. Radiant flux (watts)		Luminous flux (light-watts)		Luminous flux (lumens)
3. Radiant emission (watts)		Luminous emission (light-watts)		Luminous emission (lumens)
4. Specific radiant emission or radiant emissivity (watts per square centimeter)		Specific luminous emission or luminous emissivity (light-watts per square centimeter)		Specific luminous emission or luminous emissivity (lumens per square centimeter)
5. Radiant intensity in a given direction (watts per unit solid angle)		Luminous intensity (light-watts per unit solid angle [pyrs])		Luminous intensity (lumens per unit solid angle [candles])
6. Specific radiant intensity in a given direction, or radiance (watts per projected square centimeter per unit solid angle)		Brightness (light-watts per projected square centimeter per unit solid angle [pyrs per square centimeter])		Brightness (lumens per projected square centimeter per unit solid angle [candles per square centimeter])
7. Irradiation (watts [incident])		Illumination (light-watts)		Illumination (lumens)
8. Specific irradiation (watts [incident] per square centimeter)		Specific illumination (light-watts per square centimeter)		Specific illumination (lumens per square centimeter [phots])

9. Reflecting power = ratio of reflected to incident energy
10. Absorbing power = ratio of absorbed to incident energy
11. Transmitting power = ratio of transmitted to incident energy
12. Emitting power* = ratio of emission to that of black body at the same temperature

* The alternative equivalent endings in "ive," such as "emissive," are sometimes preferable from the standpoint of euphony.

through a template giving a spectral luminosity-curve,[1] that is, they are now illuminational quantities in the same units as used in measurements of radiation, e.g., watts; in the third column are the quantities of the second column after division by the mechanical equivalent of light;[2] that is, they are illuminational quantities expressed, as a rule, in the everyday units of illumination based on the candle.[3]

In order to illustrate clearly the pressing need for standardizing the nomenclature of radiation, I append the accompanying list (Table II)—undoubtedly far from complete—of terms which have been used in various recent textbooks on radiation and general physics.

TABLE II

Proposed in Table I	Various Textbooks
Specific radiant emission or radiant emissivity	Emissive power Specific emissivity Coefficient of radiation Emission Total energy stream
Radiant intensity	Radiation intensity Coefficient of emission Normal energy stream
Radiance	Specific intensity Brightness Radiation intensity of a surface Emissive power of a surface
Specific irradiation	Intensity
Absorbing power	Absorption Coefficient of absorption
Emitting power	Emissivity

[1] A full discussion of the relation between radiant flux and luminous flux will be found in the *Journal of the Franklin Institute*, **180**, 409, 1915, in the article "The Establishment of Photometry on a Physical Basis."

[2] 1 lumen = 0.00159 light-watt.

[3] A full discussion of the illuminational units will be found in the standard lists of the Illuminating Engineering Society and the Institute of Electrical Engineers. In details where these lists have not been followed the present table is in substantial agreement with an excellent paper by E. B. Rosa, *Bulletin of the Bureau of Standards*, **6**, 543, 1910.

For some quantities it appears from my search that most authors have no terms at all, and it is evident that the same terms are used by different authors with quite different meanings. It is only just to state that the better writers, having once defined their quantities, are likely to use them consistently, however confusing the chosen names may be. When, however, it comes to the discussion of Kirchhoff's law, which must figure in every treatment of radiation, and the statement of which is more or less stereotyped, the confusion in nomenclature has resulted in really serious confusion of ideas.

Before taking up the reasons for the choice of the various radiational terms it is perhaps advisable to make clear their use by an example which includes all except the various "powers" as follows:

Let us take a *black body* with an opening of area s; from this proceeds *radiant energy;* the amount of this given out in unit time, or the rate of emitting energy, is the *radiant emission* of this black body; this divided by s, or the rate of energy emission per unit area, is the *specific radiant emission or radiant emissivity*, which is equal to $\sigma(T^4-T_0^4)$, where σ is approximately 5.7×10^{-12} watts per degree^4, T being the temperature of the black body and T_0 that of the body which it irradiates. If we now place a radiometer of area a at such a distance T from the black-body opening that the latter is, to within the errors of measurement, a point source, and that the radiometer subtends from the black-body opening the solid angle $\Omega\left(=\frac{a}{r^2}\right)$, we can measure the *radiant intensity* of the black body *in that direction* by the quotient of the rate of reception of energy divided by Ω. This will vary from a maximum, normal to the opening, to zero when the radiometer lies in the plane of the opening. If now we divide this radiant intensity in any direction by the projected area of the opening in that direction, or by $s\cos\theta$, where θ is the angle giving the direction measured from the normal, we obtain the specific radiant intensity or *radiance* in that direction. For the black body, which obeys Lambert's law, the radiance is a constant in all directions outside the body, and is equal to $\frac{\sigma}{\pi}(T^4-T_0^4)$.

Leaving now the radiating body, consider the space between it and any body on which the radiant energy falls. Through this space radiant energy is passing at a certain rate. This rate constitutes the *radiant flux* through the space. Thus the radiant flux through the space surrounding a single radiator is equal to the radiant emission of the radiator, but radiant flux in space can be measured irrespective of whether we know what the radiator is, or in case the latter is multiple or otherwise complex in a manner to interfere with its handling by summational processes.

Somewhere in the path of the radiant flux from our black body let there be a body of area a (e.g., the radiometer assumed above). Radiant energy is incident upon this at a certain rate; this rate of reception of energy constitutes the *irradiation* of the body. The *specific irradiation* is the irradiation divided by a, the area. At any finite distance from the radiator the specific irradiation of a body will be different from point to point, and we can measure only the mean irradiation or the irradiation of an element small enough to be considered as uniformly irradiated. If the radiant energy is incident at an angle ϕ to the normal, the specific irradiation is $\cos\phi$ times the value for normal incidence. The irradiation of the body considered, owing to the assumed black-body opening, is therefore

$$\frac{s\sigma(T^4-T_0^4)}{\pi}\cdot\frac{a}{r^2}\cos\theta\cos\phi,$$

where r is the distance between the two bodies; the specific irradiation is

$$\frac{s\sigma}{\pi r^2}(T^4-T_0^4)\cos\theta\cos\phi.$$

Let us now consider this black body as a light-source. This merely means that our radiometer is to be selective, its distribution of sensibility according to wave-length being that of the eye. The light-evaluated radiant energy proceeding from the opening of the black body we call *luminous energy;* the rate at which it is given out is the *luminous emission* of the body. It is the integral

$$s\int_0^\infty J_\lambda L_\lambda d\lambda,$$

where J_λ is the specific radiant emission or emissivity for the wavelength interval $d\lambda$; L_λ is the luminosity of the normal equal-energy spectrum, in terms of the maximum as unity, or the *luminous efficiency*. This divided by s is the *specific luminous emission*, or *luminous emissivity*. *Luminous flux* and *luminous intensity* are similarly derived directly from the radiational quantities, and so on down the line. It is only when we take the further step of expressing our illuminational quantities in the ordinary units referred to the candle by dividing by the mechanical equivalent of light that we must learn new terms, namely, those for the unit of luminous flux, the *lumen*,[1] for the unit of luminous intensity, the *candle*, and for specific illumination, the *phot*. The latter, the lumen per square centimeter, proposed by M. Blondel, is the exact parallel of the watt per square centimeter by which specific irradiation is measured, and (at least in the form of its subdivision, the millphot) should take the place of the anomalous meter-candle or lumen per square meter. ("Illumination," it may be noted, is rather commonly used where "specific illumination" is meant. The illumination of a body, on all parts of which luminous energy is incident, is greater the larger the body, while the specific illumination, whether we measure it in phots or meter-candles, is not.) In the case of the black body, which has been taken as our illustration, the illumination of the surface a at the distance r is

$$\frac{L}{m}\frac{s\sigma(T^4-T_0^4)}{\pi}\frac{a}{r^2}\cos\theta\cos\phi,$$

and its specific illumination is

$$\frac{L}{m}\frac{s\sigma(T^4-T_0^4)}{\pi r^2}\cos\theta\cos\phi,$$

where L is the luminous efficiency of the black body at the temperature at which the observation is made (assumed to be calculated

[1] It would be highly desirable to use the term "lumen" for the watt of luminous flux, calling the present "lumen" the "candle-lumen," in the same manner that we designate the lumen derived from the Hefner as the "Hefner-lumen." Some of the other quantities in the second column of the table might appropriately receive names recalling worthies in photometric science, in the manner that "volt," "ampere," and "farad" do in the electrical units. The "rumford" as the rational unit of specific illumination (corresponding to the phot) would be particularly fitting.

from the spectral distribution of emission, e.g., the Planck equation, and the spectral distribution of luminosity, e.g., the equation derived by Kingsbury[1]), and m is the mechanical equivalent of light.

Taking up now the justification for the selection of names used in the radiation table, I assume that there will be little exception taken to the first three terms—"radiant energy," "radiant flux," and "radiant emission." All three of these are commonly spoken of as "radiation," but they must be differentiated as soon as any detailed discussion of the processes of radiation is undertaken. Nor should there be opposition to the last four terms, the "reflecting," "absorbing," "transmitting," and "emitting" powers, when it is pointed out that two other common ways of denominating these properties, shown in Table II, conflict with other branches of physics and with well-established usage in nomenclature. Thus the use of the word "coefficient," as in "absorption coefficient," immediately encounters the difficulty that the coefficient of absorption is used in physical optics as the fraction absorbed (and sometimes curiously enough, as the fraction transmitted) by unit thickness, which is an altogether different meaning from the one associated with absorbing power in Kirchhoff's law, for instance. The termination "ity," found in the rather frequently used "emissivity" (emitting power), is quite generally in use to indicate a property referred to *unit* mass, thickness, area, etc., as in "resistivity." "Emissivity," therefore, does not belong among the list of properties, but finds its proper place as the equivalent of "specific emission."

The greatest novelty in these suggested terms lies in the choice of the words "radiance" and "irradiation," for the radiational quantities corresponding to "brightness" and "illumination." These have not been selected from terms in use, because, speaking broadly, no one appears to have attempted to give these quantities names. In the case of "irradiation" the vast majority of writers have not been interested in what becomes of the radiant energy after it leaves the radiator. In the case of illumination the *raison*

[1] See Ives and Kingsbury, "The Mechanical Equivalent of Light as Determined from the Brightness of the Black Body," *Physical Review*, **8**, 177, 1916.

d'être of the whole science is putting the luminous energy upon something, so that not only have we a term for the rate of doing this, but this term becomes the name of the whole science. By analogy one might be led to call the rate of putting radiant energy upon something "radiation," and this usage of the word is, in fact, found in connection with the solar constant. This analogy is wrong, however, for etymologically the radiational equivalent of "illuminate" (in-luminate) is "irradiate." Strict etymological consistency would suggest that the science of illumination might more properly be denominated "lumination." It is interesting to note that all the standard dictionaries give as the first definition of "irradiate" "to throw light upon," and give as their authority the etymology together with well-supported literary usage. A dictionary is of course not the place to go for definitions of scientific terms, since the dictionary merely records the usage of scientific writers, but this appears to be a case where scientific writers have not taken advantage of a word lying ready to hand. The problem is handled by the German language in a similar manner, by the use of the prefixes *zu*, *aus*, *be*, *-strahlung*.

An objection to the employment of the word "irradiation" in this sense will occur to everyone, namely, that this word is already used for quite a different thing, the apparently larger area of a bright as compared with a dark object of the same actual dimensions. But this objection is not serious; first, because there is no great likelihood of confusion between phenomena in such different fields of optics, and second, because if the point is pressed it will be found that "irradiation" as used in physiological optics is clearly the wrong word. Etymologically it should be "eradiation," a word which, on reference to standard dictionaries (not knowing whether such a word was recognized), I find to be "a shooting out as of rays," an exact description of which is often called "irradiation."[1]

[1] "Eradiation" for radiant emission, and "elumination" for luminous emission, with the corresponding verbs "to eradiate" and "to eluminate," might claim recognition, but they are obviously impracticable because of the danger of confusion with the words having the prefix *i*. "Radiate" is used pretty generally in the sense of the suggested "eradiate," and, as noted, "radiation" for radiant emission, with probably little danger of ambiguity. For "eluminate" we already have the good word "shine." Why not call the luminous emission of a light-source its "*shine*"?

It appears, therefore, that however unusual "irradiation" may seem in the sense used in the table, it is soundly based. It will probably be granted that, in describing the condition holding in a "black" inclosure, there is conciseness and non-ambiguity in the statement that, if we are dealing with luminous flux, the *specific illumination* of all bodies therein is equal, and if we are dealing with radiant flux, the *specific irradiation* of all bodies therein is equal.

"Radiance," as a time-saving word for "specific radiant intensity," is frankly taken from the dictionary, as the appropriate synonym for "brightness."

The question may be raised in some quarters whether there is any real need for names for these quantities. Why should we not simply speak of "incident energy" and "incident energy per unit area," and so on? Apart from the service which is rendered to any science by a well-considered terminology, which should be sufficient excuse for undertaking such an arrangement as is here attempted, there is the fact to be considered that the problem of heating a body by the incidence of radiant energy is taking on technical importance. "Irradiation" may in the future figure in a manner comparable with "illumination." The measurement of radiant energy, both at the point of emission and at the point of incidence, may assume an importance comparable with that of photometry. If so, an arrangement and simplification of the terminology of radiation to fit it for technical application is inevitable.

Returning now to Kirchhoff's law, this is most frequently given, following Kirchhoff's own terminology, as a statement about the ratio of the emissive *power* of a body to its absorptive *power*. The use of the same word *power* in these two compound terms is highly objectionable, because the impression is inevitable that these two "powers" are similar properties of a body, just as are absorbing power and reflecting power. In fact, I find one writer specifically stating that they *are* similar, *since they are both functions of* λ *and* T. Actually, one is a simple dimensionless numerical coefficient, the other a rate of emitting radiant energy. The absorbing power may or may not vary with temperature. In the black body it does not. In many metals it is constant over a very large range of temperature. The radiant emissivity (the "emissive power" of Kirchhoff)

practically always varies continuously and rapidly with temperature. The two things are fundamentally different, and should not be described by similar names. The use of letters from the same case to symbolize them, as is common in the practice of representing the one by capital A and the other by capital E, is objectionable.

In accordance with the terms listed above the correct statement of Kirchhoff's law becomes: *The radiant emission of any body, due to temperature alone, is equal to the product of the absorbing power of the body, by the radiant emission of a black body, of the same dimensions, at the same temperature.*

Thus stated, Kirchhoff's law is not a relationship between two apparently similar *properties* of the *same* body (as would be a relationship between reflecting power and absorbing power), as Kirchhoff's terminology appears to indicate, but a relationship between the *behaviors* of two *different* bodies, under the same condition of temperature. A very considerable part of the service rendered by the discovery of this law is the fact that it makes possible the prediction of behavior under extreme conditions of temperature from an optical property—this property, as it happens, being one that as often as not is not variable with temperature over the region in which we are interested.

The desire, which seems to be general, to derive from Kirchhoff's law some property which is common to all bodies may be satisfied, using the terminology here suggested, by stating the simple relationship that, for any body whose radiant emission is due to temperature alone, *the emitting power is equal to the absorbing power.*

PHYSICAL LABORATORY OF THE UNITED GAS IMPROVEMENT CO.
PHILADELPHIA
November 18, 1916

THE ECLIPSING VARIABLE STAR SS CAMELOPARDALIS[1]

By R. J. McDIARMID

The variation of this star was discovered by Miss A. J. Cannon, in 1908, and announced in *Harvard Circulars No. 140* as a variable of the Algol type with a photographic range of over one magnitude, from 9.8 to 11.0.

The position of the star, $\alpha = 7^h 4^m$, $\delta = +73^\circ\ 30'$ (1900), makes it a very suitable object for observation throughout the entire year, at the latitude of Princeton. In March 1913 the writer added this star to the program of variables under observation with the sliding-prism polarizing photometer attached to the 23-inch equatorial. The period of the star was unknown at this time, as well as the nature of the variation in light. The first night's observations, March 31, 1913, gave a variation of $0^M.50$, as the star was then in primary eclipse. During the two years' observation the variation never exceeded $0^M.60$ and this was very puzzling, as it was announced with a variation of over one magnitude. This discrepancy was not cleared up until after the true period was determined, and it was found that the difference in photographic and visual range was due to the color-index of the star.

The period of this star was very difficult to determine owing to the length of the eclipses, which last over 21 hours. In the autumn of 1914 Professor Pickering of Harvard very kindly sent me 30 photographic observations taken at and near primary minimum, extending as far back as 1896. These observations were quite well satisfied with a period of 4.825 days, and a range of $1^M.2$; a note was added that in some cases the measures were difficult to make and were perhaps in doubt, owing to the adjacent bright star B.D.$+73^\circ 366$. The variable is referred to in *Harvard Circular No. 140* in the following terms: "This star follows $+73^\circ 366$, magn. 7.5, 24^s, north $0'.5$". The period $4^d.825$ applied to the

[1] Paper read at nineteenth meeting of the American Astronomical Society, *Popular Astronomy*, **24**, 668, 1916.

visual observations was found to be too long. When the data furnished by the visual and photographic observations were combined, more weight being assigned to the visual, the period $4^d19^h47^m6^s.4$ was determined, which satisfied both series of observations very well. The photographic observations used in this case, as well as in that of TX Cassiopeiae[1] and T Leonis Minoris, proved to be of extreme value.

The observations made with the Princeton photometer extend over a considerable period, from March 31, 1913, to December 10, 1915. In all, 10,992 measures were made by comparing SS Camelopardalis with the star whose co-ordinates[2] are $\alpha = 7^h2^m21^s.8$, $\delta = 73°\ 12'27''$ (1900), and visual magnitude 11.14. The magnitude of the comparison star was found by comparing it with B.D.$+73°366$, whose magnitude is given in *Harvard Annals*, **45**, as 7.70.

After correction for light-equation, the 687 observations (of 16 settings each) were grouped into normals according to phase, with the finally adopted period; each normal was the weighted mean of five observations, except in three or four cases; the weights were assigned in accordance with notes taken at the time of observation (maximum weight being 5).

The observed curve thus formed presented some unusual characteristics for an eclipsing variable. The depth of the primary eclipse is $0^M.57$, and that of the secondary is $0^M.15$. The primary eclipse lasts 21 hours, and there is a period of constant brightness of 7 hours at minimum phase. The secondary is of the same duration, but the interval from primary to secondary is shorter than that from secondary to the following primary, showing that the orbit is eccentric. The uneclipsed portions of the light-curve show a slight ellipticity of the stars, but, in addition, the portion of the curve between primary and secondary is about $0^M.05$ lower than that between secondary and primary. The observations of these two branches were taken at different seasons, and it might

[1] *Astrophysical Journal*, **42**, 412, 1915.

[2] The co-ordinates of the comparison star were computed for 1900 from the data furnished in the *Astrographic Catalogue* of the Greenwich Observatory, **2**, 126, Plate 3008; number of star on plate, 2989.

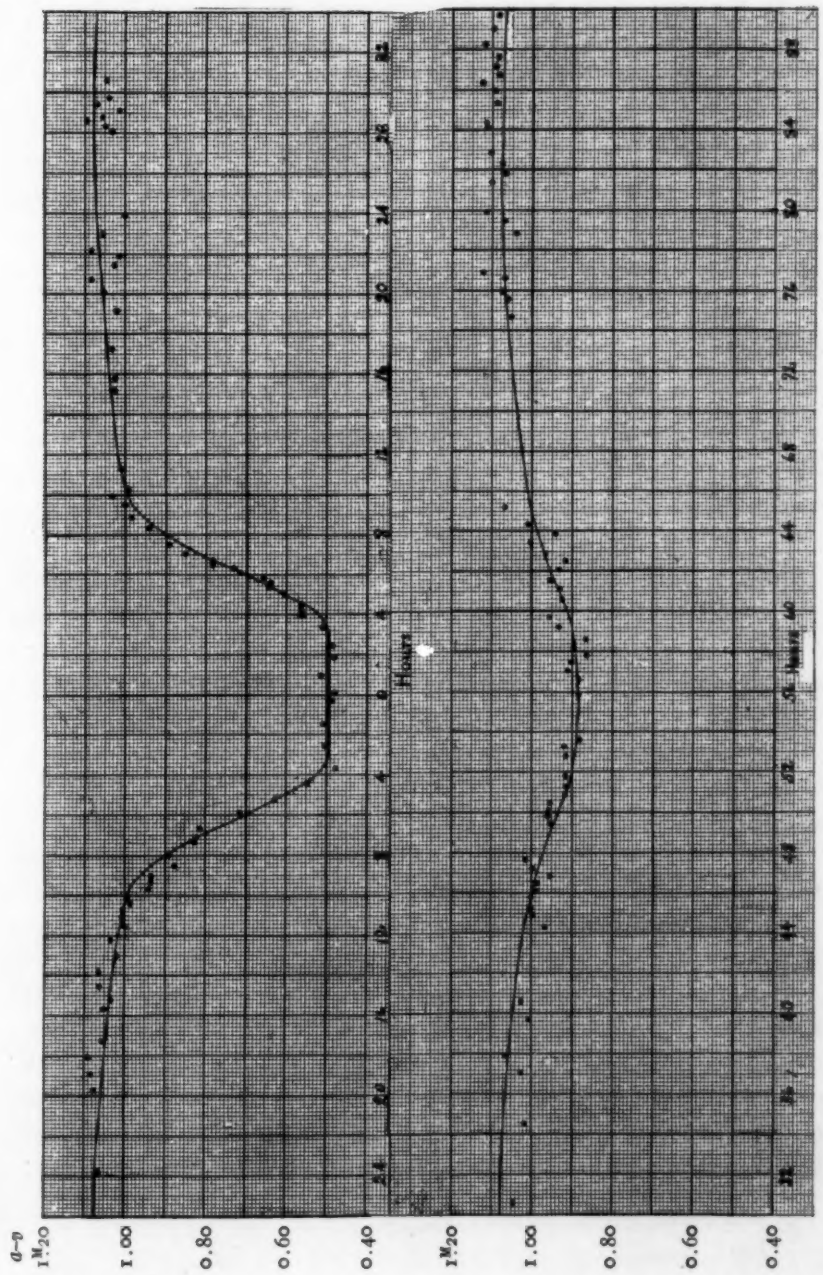
be possible that the difference in "level" is due to the difference in sensitiveness of the observer's eye; this, however, was never found to be the case in the four[1] other systems previously worked up. The comparison star might itself be a variable, as in the case of TX Cassiopeiae;[1] this did not seem to be the case, as the comparison star was compared with B.D.+73°366 on several different nights; the measures thus made showed no greater range than that corresponding to the probable error usually found. It would, therefore, appear that this difference is real, and that SS Camelopardalis is fainter after the primary eclipse than before it. This, however, is the opposite of what might be expected, since after primary minimum the components are near periastron, which is associated with unusual brightness in other systems; but the observational evidence appears to be conclusive. To eliminate this influence, the ellipticity-constant was determined from the mean of the branches.

The "rectified" depths of the two minima, after correction for ellipticity, are $0^M.486$ and $0^M.082$, and the loss of light in the two cases $1-\lambda_p$ (primary) $=0.3610$ and $1-\lambda_s$ (secondary) $=0.0727$. The primary eclipse is obviously total, since the intensity remains constant at minimum for over 7 hours. Hence from the relation for uniform disks, $k^2=\frac{1-\lambda_s}{\lambda_p}$, the value of k, the ratio of the radii of the stars, is 0.335. Since the branches of the observed curve at primary minimum seem to show a slight asymmetry, the two branches were combined in one, and this curve was used in the solution, taking $k=0.335$ and proceeding in the usual way.[2]

The elements resulting from the solution were computed in the manner outlined in *Astrophysical Journal*, **36**, 404, 1912, and are given in Table III. The residuals for primary and secondary were read off from the theoretical curve, and are given in the table of observations under $(O-C_m)$. The probable error of a normal point for primary is $\pm 0^M.016$ and for secondary, $\pm 0^M.018$.

[1] *Astrophysical Journal*, **42**, 412, 1915.

[2] *Ibid.*, **35**, 315, 1912; **36**, 54 1912.



Mean light-curve of SS Camelopardalis

FIG. 1

In the case of the solution which considers the stars darkened to zero toward the edge, the primary eclipse was total and the secondary annular, as in the previous solution. The secondary was assumed slightly deeper, viz., $1-\lambda_s=0.0850$. Then from the relation

$$Q(ka_0)=\frac{1-\lambda_2}{a_0-(1-\lambda_1)}$$

where

$$1-\lambda_2=0.0850$$
$$1-\lambda_1=0.3610$$
$$a_0=1.00 \text{ for total eclipse}$$

we obtain

$$Q(ka_0)=0.133.$$

From Table V,[1] for central annular eclipse $a_0''=1+x$, $k=0.307$; for grazing annular eclipse $a_0''=1.00$, $k=0.358$. After several trials were made the best "fit" was found by adopting

$$a_0''=1+0.25x \text{ and } k=0.345.$$

The primary curve was now computed for $k=0.345$ by means of Table IIx,[2] and the curve for secondary eclipse, for $k=0.345$ and $a_0''=1+0.25x$, by use of Table Iy.[3] The corresponding "darkened" elements are given in Table III. The residuals for primary and secondary eclipse are given in Table I in the column headed $O-C_d$. The probable error of a normal point is $\pm 0^M.016$; and for secondary, $\pm 0^M.018$.

The tabulated densities in both cases are corrected for polar flattening as well as for probable difference of the masses of the brighter and fainter components. The hypothetical parallax was computed, and also the galactic co-ordinates.

The plot of the photographic observations on the new period gives the brightness as $9^M.8$ at constant light and $10^M.86$ at primary minimum, giving a loss of light of $1^M.06$. Adopting the elements previously obtained, which after rectification would probably be about $0^M.95$ and $1-\lambda_p$ (photographic)$=0.583$, the light of the bright star is 0.583 of the total, that of the faint one being 0.417;

[1] *Astrophysical Journal*, **36**, 394, 1912.

[2] *Ibid.*, 245, 1912.

[3] *Ibid.*, 390, 1912.

hence J_b/J_f (ratio of surface brightness) = 12.2 for the uniform solution, and for the darkened solution, 11.7.

TABLE I

OBSERVATIONS SS CAMELOPARDALIS

PRIMARY MINIMUM

Normal No.	Phase	Mag. Diff. v-a	O−C$_u$	O−C$_d$	Normal No.	Phase	Mag. Diff. v-a	O−C$_u$	O−C$_d$
1.....	−12^{h}11^{m}3	1.035	+0.015	+0.015	24.....	+ 1^{h}51^{m}1	0.480	−0.012	−0.012
2.....	11 26.2	1.004	− .010	− .007	25.....	2 26.5	0.481	− .011	− .011
3.....	11 2.4	1.008	− .005	− .003	26.....	3 21.6	0.510	+ .028	+ .012
4.....	10 39.6	1.007	+ .004	.000	27.....	3 57.7	0.554	+ .036	+ .037
5.....	10 22.5	0.987	− .018	− .018	28.....	4 13.9	0.503	+ .028	+ .033
6.....	10 3.9	0.996	− .005	− .002	29.....	4 32.7	0.564	+ .005	.000
7.....	9 40.6	0.940	− .050	− .050	30.....	4 59.9	0.605	.000	.000
8.....	9 21.3	0.936	− .045	− .045	31.....	5 22.7	0.644	+ .010	+ .006
9.....	8 58.8	0.932	− .035	− .028	32.....	5 37.2	0.639	− .018	− .020
10.....	8 32.1	0.878	− .052	− .052	33.....	5 50.9	0.655	− .022	− .025
11.....	8 00.1	0.892	.000	− .010	34.....	6 06.0	0.716	+ .010	.000
12.....	7 21.6	0.829	− .005	− .008	35.....	6 18.0	0.730	.000	− .004
13.....	6 40.3	0.815	+ .035	+ .035	36.....	6 30.2	0.784	+ .034	+ .034
14.....	5 53.7	0.714	+ .014	+ .024	37.....	6 43.6	0.784	+ .014	.000
15.....	5 12.3	0.629	+ .005	+ .003	38.....	7 03.6	0.851	+ .050	+ .030
16.....	4 24.9	0.548	.000	− .005	39.....	7 31.4	0.892	+ .040	+ .042
17.....	3 39.5	0.477	− .015	− .033	40.....	8 21.2	0.943	+ .018	+ .016
18.....	2 31.7	0.504	+ .012	+ .012	41.....	8 51.0	0.987	+ .037	+ .030
19.....	1 26.4	0.509	+ .017	+ .017	42.....	9 32.0	1.003	+ .018	+ .018
20.....	0 48.9	0.495	+ .003	+ .003	43.....	9 57.7	1.034	+ .036	+ .040
21.....	−0 17.0	0.485	− .007	− .007	44.....	10 17.6	0.992	− .010	− .008
22.....	+0 6.0	0.480	− .012	− .012	45.....	11 14.1	1.008	−0.005	−0.007
23.....	0 59.5	0.512	+0.030	+0.030	.....	.....	.....	.....	.....

SECONDARY MINIMUM

70.....	+44 17.2	0.966	−0.046	−0.050	89*.....	+55 52.4	0.882	−0.014	−0.003
71.....	44 56.6	0.998	− .010	− .014	90.....	56 38.3	0.884	− .012	− .005
72.....	45 19.8	1.001	− .006	− .005	91.....	57 3.5	0.910	+ .014	+ .022
73.....	45 47.2	1.006	.000	.000	92.....	57 30.7	0.901	+ .005	+ .012
74.....	46 9.4	0.986	− .008	− .010	93.....	57 50.2	0.861	− .035	− .035
75.....	46 28.3	0.984	− .005	− .008	94.....	58 11.1	0.899	+ .003	+ .004
76.....	46 50.0	0.952	− .032	− .036	95.....	58 38.9	0.866	− .030	− .040
77.....	47 10.1	0.997	+ .015	+ .015	96.....	59 12.6	0.932	+ .030	+ .030
78.....	47 38.3	1.014	+ .041	+ .036	97.....	59 41.3	0.958	+ .048	+ .050
79.....	49 26.1	0.952	+ .012	+ .007	98.....	60 34.6	0.923	.000	+ .002
80.....	49 49.6	0.960	+ .030	+ .020	99.....	61 6.9	0.930	− .005	− .004
81.....	50 7.9	0.958	+ .032	+ .028	100.....	61 35.4	0.953	+ .012	+ .010
82.....	50 27.6	0.957	+ .030	+ .030	101.....	62 3.3	0.931	− .016	− .024
83.....	50 52.8	0.918	+ .005	+ .000	102.....	62 31.8	0.916	+ .042	+ .048
84.....	51 15.1	0.915	+ .010	+ .005	103.....	62 57.1	0.967	.000	− .005
85.....	51 49.1	0.913	+ .015	+ .013	104.....	63 26.8	1.002	+ .030	+ .024
86.....	52 48.7	0.916	+ .020	+ .022	105.....	63 55.5	0.939	− .042	− .043
87.....	53 16.4	0.920	+ .026	+ .028	106.....	64 23.1	1.009	+ .017	+ .017
88*.....	53 35.3	0.881	−0.015	−0.009	107.....	65 12.2	1.068	+0.068	+0.065

*Zero phase of secondary is 55^{h}24^{m}0.

The primary eclipse is total; this is certain, since the light remains constant for 7 hours. Hence, knowing the magnitude of the system, we can readily determine the magnitude of each star. The visual and photographic magnitudes resulting are given in

Table III. In this system we meet again the combination, already noticed in other eclipsing binaries,[1] of a small, relatively dense

TABLE II

UNECLIPSED OBSERVATIONS

PRIMARY TO SECONDARY				SECONDARY TO PRIMARY			
Normal No.	Phase	Mag. Diff. (a−v)	O−C	Normal No.	Phase	Mag. Diff. (a−v)	O−C
46....	+15h12m4	1.027	−0.012	108	+74h40m2	1.052	−0.010
47....	15 49.3	1.024	− .015	109	75 38.5	1.060	− .008
48....	17 15.0	1.032	− .012	110	76 00.0	1.077	+ .008
49....	19 12.3	1.020	− .028	111	76 35.1	1.070	.000
50....	20 6.3	1.052	− .005	112	76 54.5	1.022	+ .052
51....	20 44.0	1.084	+ .022	113	78 53.8	1.041	− .035
52....	21 26.2	1.026	− .035	114	79 29.9	1.069	− .005
53....	21 52.8	1.013	− .046	115	79 56.2	1.115	+ .040
54....	22 12.6	1.083	+ .018	116	81 23.3	1.102	+ .025
55....	23 0.1	1.056	− .015	117	81 50.4	1.067	− .010
56....	23 56.4	1.000	− .070	118	82 20.5	1.077	+ .000
57....	28 4.7	1.032	− .045	119	82 52.7	1.104	+ .026
58....	28 21.4	1.049	− .026	120	84 20.6	1.115	+ .041
59....	28 35.7	1.095	+ .015	121	85 20.6	1.089	+ .013
60....	28 48.7	1.055	− .023	122	85 59.4	1.092	+ .022
61....	29 8.5	1.012	− .066	123	86 22.9	1.126	+ .056
62....	29 26.5	1.069	− .010	124	86 46.8	1.084	+ .014
63....	29 44.7	1.040	− .035	125	87 12.7	1.091	+ .022
64....	30 36.2	1.046	− .032	126	+87 37.6	1.086	+ .016
65....	34 32.8	1.016	− .054	127	−27 28.8	1.120	+ .051
66....	37 6.0	1.024	− .036	128	26 44.5	1.098	+ .030
67....	37 55.1	1.065	+ .010	129	26 2.5	1.084	+ .016
68....	39 43.8	1.007	− .043	130	23 48.6	1.065	− .005
69....	40 36.9	1.023	−0.020	131	19 44.1	1.078	+ .020
.....	.....	.....	.....	132	18 53.8	1.085	+ .030
.....	.....	.....	.....	133	18 5.0	1.091	+ .042
.....	.....	.....	.....	134	17 17.1	1.056	+ .008
.....	.....	.....	.....	135	15 41.5	1.052	+ .010
.....	.....	.....	.....	136	15 11.1	1.035	− .005
.....	.....	.....	.....	137	14 32.4	1.062	+ .027
.....	.....	.....	.....	138	13 46.2	1.065	+ .035
.....	.....	.....	.....	139	−12 58.3	1.020	+0.005

star, of greater surface brightness, and presumably hotter, with a large star of low density and surface brightness, redder color,

[1] *Contributions from the Mt. Wilson Solar Observatory*, Nos. 64 and 68; *Astrophysical Journal*, **36**, 368, 1912; **37**, 204, 1913.

and doubtless lower temperature. The unusual feature is that, visually, the larger star, although fainter per unit area in a ratio of 1:5, gives the greater amount of light. Photographically the reverse is true; the photographic plate is affected chiefly by the light of the smaller star, which is 12.2 times brighter per unit area than the larger star. The spectral type of the system is given as F?; if this is the case, the smaller star may well be of Class A and the larger of Class G. On this assumption the hypothetical parallax comes out 0".0007 from the data for the small bright star, and 0".0011 from those for the large faint star.

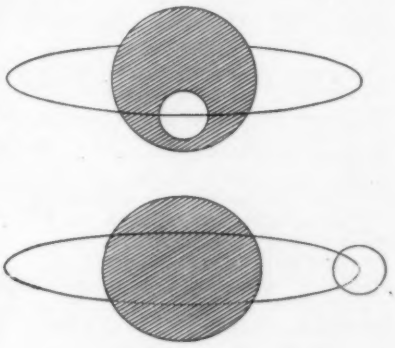


FIG. 2

After the completion of the discussion of the observations, a paper by R. S. Dugan on the variable RV Ophiuchi was published.[1] The light-curve of this star has characteristics similar to those of SS Camelopardalis. The difference in brightness between the uneclipsed portions of the curve is almost exactly represented by a sine term. Professor Dugan explains it as due to a difference of brightness between the advancing and following sides of the brighter star. This assumption was applied to SS Camelopardalis, and it was found that by using a term 0.020 sin θ (θ being the orbital longitude) the residuals throughout the curve were greatly reduced. In this case the system is brightest before principal minimum, but, since the star then eclipsed gives out less light than the other, this is actually the case when the brighter of the two components (though not the one of greater surface brightness) is approaching us.

SUMMARY

1. SS Camelopardalis is an eclipsing variable. The elements for the principal minimum are: J.D. 2,420,842.594+4^d.82438= 1915 December 10, 14^{h}21^m G.H.M.T.+4^{d}19^{h}47^{m}6^s.4.

[1] *Astrophysical Journal*, 43, 130, 1916.

2. The range from maximum to principal minimum is $0^{M}.57$, and to the secondary minimum is $0^{M}.15$. The stars are distinctly ellipsoidal, and when correction is made for this the depths of the minima become $0^{M}.485$ and $0^{M}.082$ respectively. The brightness

TABLE III

RESULTS

Elements of System	Uniform	Darkened
Maximum radius of small star.....a_2	0.150	0.153
Minimum radius of small star.....b_2	0.136	0.144
Maximum radius of large star.....a_1	0.446	0.444
Minimum radius of large star.....b_1	0.406	0.417
Ratio of the radii of the two stars.....k	0.335	0.345
Ratio of the axes of the spheroidal stars.....$1+\frac{1}{2}\epsilon$	1.088	1.054
The least apparent distance of centers ..... $\cos i$	0.2024	0.2305
Inclination of orbit.....i	78°.3	76°.0
Eccentricity of orbit.....$e \cos \omega$	−0.03	.....
Maximum percentage loss of light at primary minimum.....$\alpha_{o,p}$	1.00	1.00
Maximum percentage loss of light at secondary minimum.....$\alpha_{o,s}$	1.00	*1.096
The light of the bright star (visual).....L_b	0.6397	0.6397
The light of the faint star (visual).....L_f	0.3603	0.3603
Ratio of the surface brightness of the two stars (visual).....J_b/J_f	5.02	4.74
The light of bright star (photographic).....L_b	0.583	.....
The light of faint star (photographic).....L_f	0.417	.....
Ratio of surface brightness of the two stars (photographic).....J_b/J_f	12.2	11.7
Most probable density of smaller star.....ρ_b	0.094	0.078
Most probable density of larger star.....ρ_f	0.0050	0.0046
Hypothetical radius of brighter star in terms of solar radius.....	2.32	2.32
Hypothetical parallax		0".0009
Distance in light-years from the galactic plane		1700
Distance in light-years projected on galactic plane		3200
Visual magnitude of system		10.15
Visual magnitude of big star (end on)		10.65
Visual magnitude of small star (end on)		11.26
Photographic magnitude of system		9.9
Photographic magnitude of big star (end on)		10.86
Photographic magnitude of small star (end on)		10.48

*$\alpha_{o,s}$ Expressed in terms of the light lost at internal tangency.

is greater by about $0^{M}.05$ halfway between secondary and primary than midway between primary and secondary. The principal eclipse lasts 21 hours and is total for 7 hours; the secondary is annular. The photographic range at principal minimum is about $0^{M}.95$.

3. The system consists of a large red star of low surface brightness and a smaller white star of a little more than one-third of the diameter of the other, but of five times the surface brightness of the large star visually and 12 times its surface brightness photographically, so that the smaller star is visually the fainter and photograpically the brighter of the two. Both stars are ellipsoidal, the ratio of the longer to the shorter axis in the equatorial plane being about 1.06.

4. The density of the large red star is about 1/200, and that of the smaller white star about 1/12 that of the sun. The combined spectrum is recorded as F?. It is not improbable that the small star is of spectral type A and the larger of type G or redder.

OTTAWA, CANADA
September 1916

THE MINIMUM RADIATION VISUALLY PERCEPTIBLE

By HENRY NORRIS RUSSELL

In a recent paper[1] Ives has shown that the least quantity of radiant energy capable of producing the sensation of light is much smaller than had previously been supposed, the error of the earlier estimates arising from the treatment of all radiations within the visible spectrum as if they were equally efficient in producing the sensation of light, which is extremely far from the truth. The numerical results given in his paper depend, however, upon four factors, two of which were well determined, while the other two could be but roughly estimated in the absence of sufficient data. It is the purpose of the present communication to supplement Dr. Ives's work by the introduction of improved values for these latter factors, derived partly from material inaccessible to him and partly from unpublished observations.

The well-determined factors are the mechanical equivalent of light and the stellar magnitude of the standard candle. The former may be taken as 1.59 ergs per second per square centimeter,[2] for light of intensity one meter-candle, and wave-length 0.55 μ (corresponding to the maximum visual sensation for equal energy). The meter-candle, according to the mean of several fairly accordant determinations,[3] is of stellar magnitude -14.18. It follows that a source emitting light of wave-length 0.55 μ, and appearing like a star of magnitude 6.0, would furnish energy at the rate of 1.35×10^{-8} ergs per second per centimeter. These data are taken unchanged from Ives's discussion.

The uncertain factors are the diameter of the pupil of the eye when accommodated to complete darkness, and the stellar magnitude of the faintest visible objects.

Upon the first of these points the only really reliable information appears to be contained in a brief paper by W. H. Steavenson[4]

[1] *Astrophysical Journal*, **44**, 124, 1916.

[2] Ives, Coblentz, and Kingsbury, *Physical Review*, **5**, 269, and **6**, 319, 1915.

[3] Russell, *Astrophysical Journal*, **43**, 129, 1916.

[4] *Journal of the British Astronomical Association*, **26**, 303, 1916.

which reached this country just after Dr. Ives's paper was written. Measurements were made by photographing the eyes, accommodated to complete darkness, by flashlight, since the presence of light strong enough to permit of direct measurement caused a decided contraction of the pupil. The mean diameter for five healthy men was found, under these conditions, to be 0.336 inch, or 8.5 mm, with an average deviation from the mean of only 3 per cent. This makes the area of the pupil 0.57 sq. cm—just double the value resulting from Ives's estimate of diameter 6 mm, and nearly three times that derived from the conventional figure of "one-fifth of an inch."

More important is the second question. There is abundant evidence to show that the visibility of faint stars is greatly affected by the diffused light of the sky. How great this influence is, even for the brighter stars, is often accidentally illustrated when a star happens to be seen through a gap in the foliage of a tree, upon a relatively dark background. To the writer, at least, it appears so much brighter than usual under these circumstances as to make immediate identification difficult.

A very valuable series of observations of the visibility of stars upon a dark background has been made by H. D. Curtis.[1] Faint stars were observed through a small hole in a large screen attached to the remoter end of the twelve-inch telescope, and the reality of the observation was controlled with the telescope. Though the faintest star seen directly upon the background of the sky was of magnitude 6.53, stars of magnitude 7.2–7.4 were easily seen upon the dark background, four others of magnitudes between 7.9 and 8.1 were usually seen with little difficulty, two of magnitude 8.3 were "seen with difficulty," and a star of magnitude 8.9 was "glimpsed at intervals, very doubtful." It appears, therefore, that if seen upon a perfectly dark background the limit of visibility for a stellar point must be not far from the magnitude 8.5—especially when it is remembered that the dome of an observatory, under ordinary working conditions, is by no means perfectly dark, even in the absence of artificial lights.

[1] *Lick Observatory Bulletin*, **2**, 67, 1901. The magnitudes of some of the stars have been replaced by more recent Harvard photometric determinations.

Further evidence bearing upon this matter is found in certain observations by the writer, part of which have been published.[1] If a room is completely darkened except for a small part of one window, less than a square foot in area, and the light of a star, shining through this aperture, is received on a white screen on the opposite side of the room the faint luminous patch may be easily observed if the eye has become thoroughly accommodated to darkness. The light of the first-magnitude stars is unmistakable, and that of Sirius really conspicuous; but in this connection the following transcript from the notebook is of interest. "Sky near Sirius much brighter than paper illuminated by Sirius-light, so much so that it dazzles the eye, appearing as a dark spot on the Sirius-lit paper, after a glance at the window. To see the fainter lights to advantage, it is necessary to remain in the dark for at least half an hour, and to avoid looking at anything as bright as Sirius-light for a few minutes before observing." Under favorable conditions, the illumination of the paper by stars fainter than the second magnitude was recognized, the faintest stars whose light was certainly identified being of magnitudes 2.8 and 2.9. As these stars were at a moderate altitude, it is probable that the illumination of a white surface by a star of the third magnitude in the zenith would be distinctly perceptible.

The following observations, which have not previously been published, are copied from the notes written at the time:

February 8, 1902. Investigations on *minimum visibile*. Two approximately circular disks of white cardboard were pinned on black cloth, and the greatest distances at which they could be seen, when illuminated by starlight, marked with pins on a tape measure. Disk A (larger) maximum distance, light from Sirius, 39 inches. Disk B (small) maximum distance, light from Sirius, 20½ inches. Disk A, light from Rigel, maximum distance 16½ inches. The disks lost all definite outline at less than half these distances and appeared as faint, ill-defined luminous spots, like the old luminous match-safes. Diameter of disk A, 0.45 inch, of disk B, 0.80 inch.

From these data the *minimum visibile* may be calculated in terms of stellar magnitude as follows. The disks were illuminated and viewed nearly at right angles to their own plane. If the radius of

[1] "Shadows Cast by Starlight," *Popular Astronomy*, **10**, 242, 1902.

the disk is r, the distance of the observer's eye R, and if it is assumed that the disk reflects diffusely the fraction A of the incident light, in accordance with Lambert's law, the light received by the eye from the disk should be to that received directly from the star in the ratio $A\frac{r^2}{R^2}$: 1, to which corresponds a difference of magnitude amounting to 5 log $R/r - 2.5$ log A. If the value of A for the white cardboard is taken roughly as 0.8, the last term is +0.25, and the three observations give (correcting the obvious interchange of A and B at the end of the original note):

Star	Disk	Mag. of Star	R	r	5 log R/r	*Minimum Visibile*
Sirius	A	−1.58	39	0.40	9.95	$8^{M}.6$
Sirius	B	−1.58	20.5	0.225	9.80	8.5
Rigel	A	+0.34	16.5	0.40	7.95	8.5

The observations, which from the nature of the measures must have been quite independent, agree better than might have been expected. The resulting *minimum visibile*, $8^{M}.5$, is in entire agreement with that found by the Lick observers, and is ten times fainter than the ordinarily assumed limit for the visibility of a star against the relatively brilliant background of the sky.

The amount of energy which would enter the eye from a light-source of maximum efficiency, and of magnitude 8.5 is, according to the foregoing, $1.35\times10^{-8}\times0.57\times0.10$, or 7.7×10^{-10} ergs per second—which may be taken as the best available approximation to the true *minimum visibile*. This is just one-fifth of the value found by Dr. Ives, as follows at once from the facts that the values here assumed for the area of the pupil and the faintest visible light are respectively twice and one-tenth of those adopted by him.

The faintest visible illumination of a surface (by a third-magnitude star) corresponds to 1.3×10^{-7} meter-candles, and, for light of the maximum efficiency, to an energy-density of 2.1×10^{-7} ergs per square centimeter per second. Since there must be some losses, both on reflection at the surface and in passing through the eye, it follows that a sensible stimulation of the retina can be produced by an energy-density less than 2×10^{-7} ergs per square

centimeter per second at the surface of the retina itself, provided that this stimulus is applied over a sufficiently large area (which, to make the incident energy equal to the minimum previously given, must have an apparent angular diameter of at least 10°). Ten times this illumination, corresponding to the light of a star of magnitude 0.5, is conspicuous to the fully rested eye, so that the "shadow bands" caused by the irregularities of density in the earth's atmosphere can be seen on the screen under favorable conditions.

The results here given are subject to possible corrections for the difference in area of the luminosity-curve for high and low intensity, but Dr. Ives has shown that these are probably small. The greatest outstanding uncertainty is that of the observational determinations of the *minimum visibile*. According to the present estimate, the minimum perceptible radiation corresponds to the reception by the eye of about 200 elementary quanta of radiation per second, or, in ordinary units, of *one erg in forty years*, or one gram-calorie in 1700 million years.

Further investigations of this question by laboratory methods would be of great interest, especially if an attempt was made to determine how long a time was required for such very faint light to produce a perceptible sensation.

PRINCETON UNIVERSITY OBSERVATORY
December 11, 1916

Minor Contributions and Notes

ON De LURY'S HYPOTHESIS AS TO EFFECT OF HAZE ON MEASURES OF SOLAR ROTATION[1]

R. E. De Lury suggests[2] that certain variations of results occurring in spectroscopic measurements of solar rotation are probably due to the superposition, on the spectrum of a definite point of the sun's image, of the spectrum of scattered light from the sun as a whole. He mentions various sources of the scattered light, including atmospheric haze, instrumental stray light, diffraction-effects, bad seeing, and possibly dust particles in space. He computes that certain results published by St. John, Adams, and Ware indicate an overlapping spectrum of the haze of about 9 ± per cent at Mount Wilson for observations of 1914–1915 in the region λ 5200.

The work of the Smithsonian Astrophysical Observatory has yielded measurements bearing on this point. In the years 1904–1907 at Washington, and 1913–1916 on Mount Wilson we observed at various wave-lengths the distribution of light along the diameter of the solar image.[3] If De Lury is right about the amount of the stray light, our work on the distribution of solar radiation would be worthless, as well as that of the observers of the solar rotation. It is worth noting that our Washington results agree closely with our Mount Wilson work and differ from it in the sense contrary to what would be expected if both were vitiated by stray light, and Washington work most affected owing to less favorable location.

In this work, having set the spectrobolometer at the desired wave-length, we set the solar image a little beyond the slit and inserted the shutter. Then we stopped the coelostat clock, and as the image drifted toward the slit we opened the shutter.

[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] *Astrophysical Journal*, **44**, 177, 1916.

[3] See *Annals*, **2** and **3**, and *Smithsonian Misc. Coll.*, **65**, No. 4, 1915.

At the central position we closed the shutter for 10 seconds, and after the image had drifted entirely off the following limb we inserted the shutter again. Thus the record runs: shutter, sky, sun, shutter, sun, sky, shutter. If now on the automatic bolographic record a straight edge is laid on the trace of the three shutter positions, the relative intensities of sky and sun are measureable. It is to be noted that "sky" means all stray light from all the sources enumerated above. As our telescope is nearly similar to that used by St. John, Adams, and Ware, excepting as their silvered mirrors may have had better or worse polish than ours, the results we obtained should give a fair test of the matter in question.

As to the results obtained in Washington work I quote: "From the examination of numerous records it appears that the total deflection produced by the sky radiation and all other causes combined, at a point 1.5 per cent of the radius outside the computed width of the disk, is certainly less than 1 per cent of the deflection at the centre of the disk."[1] (At λ 5200 the edge has about 50 per cent as great intensity as the center of the disk.)

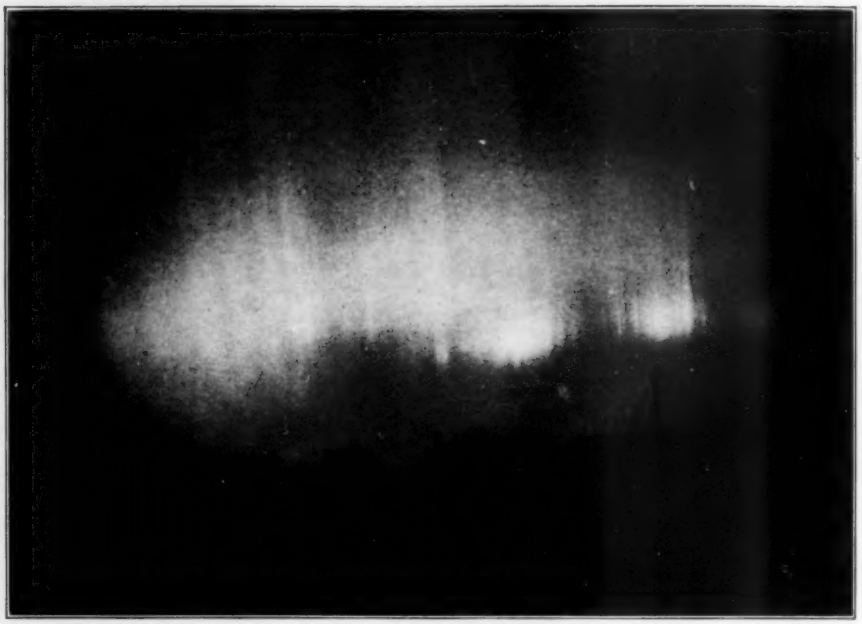
As to the results obtained on Mount Wilson, on account of the better definition and less intensity of the sky-light there, the inspection of numerous plates does not indicate appreciable intensity at a point 1.5 per cent of the radius outside the limb. So far as can be measured, there is no difference between the shutter and the sky. I feel safe in saying that the stray light there is at any rate much less than 1 per cent of the sunlight at the limb for λ 5200.

From these results I conclude that, with the long-focus telescopes used, and for good observing days, neither at Washington nor (much more) at Mount Wilson does the spectrum of a point of the sun's image at λ 5200 derive more than 1 per cent of its intensity from all the light scattered from positions distant more than 1.5 per cent of the solar radius. Experts will draw their own conclusions as to what influence scattered light of such intensity may produce on measurements of solar rotation.

Our work has been done at high sun, i.e., zenith distances almost always less than 50°. We have not observed on very hazy days,

[1] *Annals*, 2, 223.

PLATE I



AURORAL CURTAINS. 1916 October 13. 9^h34^m G.M.T.
Exposure 3 seconds

AURORAL ARCH. 1916 October 13. 9^h57^m G.M.T.
Exposure 10 seconds

such as would be obviously unfit for our investigations of the solar constant. It is possible that if observations were made soon after sunrise, or through clouds, or on extremely hazy days, the superposed spectrum would reach such values as are suggested by De Lury. Perhaps observers of solar rotation may find it a useful precaution to expose a plate to light from just within and just without the solar limb, in such a manner as to get a rough indication of the intensity of stray light on each day when the solar rotation is observed.

C. G. ABBOT

SMITHSONIAN ASTROPHYSICAL OBSERVATORY
December 18, 1916

PHOTOGRAPHS OF THE AURORA BOREALIS

A very fine aurora was seen in the northern sky at Christiania on October 13, 1916. The photographs reproduced here (Plate I) were taken with the camera used during my expedition to Bossekop in 1913 for observing the aurora. It had a kinematograph lens of 25 mm aperture and 50 mm focal length. "Ultra rapid" plates from Hauff of Feuerbach were used. Enlarged positives were first made from the small original negative; new negatives were made from these, and the illustrations are reproduced from prints taken from these enlarged negatives.

Some of the stars in Ursa Major may be seen above the aurora in the lower picture, and in the foreground of both pictures the trees and buildings of the town may be discerned. The photographs were taken by myself from the roof of the Astronomical Observatory.

CARL STÖRMER

CHRISTIANIA
November 1916

NOTICE TO SUBSCRIBERS AND CONTRIBUTORS

The war has greatly increased the difficulties of the publication of scientific journals. This is peculiarly true of the *Astrophysical Journal*, inasmuch as three-fifths of its subscribers before the war were in Europe or in countries other than the United States. The difficulty in renewing subscriptions from abroad in these troublous times has diminished the income, while the greatly increased cost of paper has unduly enhanced the expense of manufacture of the *Journal*. Even in time of peace the income has not been over one-half of the expense of production of the *Journal* in the style that has been followed for the last forty-four volumes, but the necessary subsidy has been generously maintained by the Trustees of the University of Chicago.

Hoping that the unfavorable conditions are temporary, the Editors seek to avoid the necessity of an increase in the subscription price; but we shall be obliged for the present to restrict the size of the numbers to about 64 pages and to limit the illustrations to those actually necessary. (It may be appropriate to mention that authors frequently furnish the blocks for their illustrations.) We ask our contributors to express themselves briefly and to avoid expensive tabular composition where possible. We are further obliged, for the present, to suspend the privilege of free reprints for articles received after this notice is issued. Reprints will be supplied at cost, which may be estimated as follows:

	25	50	100	150	200	250	300	350	400	500	Additional 100
4 pages.....	$1.30	$1.40	$1.60	$1.80	$2.00	$2.20	$2.40	$2.60	$2.80	$3.20	$0.40
8 pages.....	1.75	1.90	2.20	2.50	2.80	3.10	3.40	3.70	4.00	4.60	0.60
16 pages.....	2.80	3.20	4.00	4.60	5.20	5.80	6.40	7.00	7.60	8.80	1.20
Plates*.....	0.80	0.90	1.10	1.30	1.50	1.70	1.90	2.10	2.30	2.80	0.60
Covers.....	1.00	1.20	1.60	2.00	2.40	2.80	3.20	3.60	4.00	4.80	0.80

*One double plate equals two single.

THE EDITORS